A

TREATISE

OF SUCH

Mathematical Inſtruments

As are uſually put into a

PORTABLE CASE,

Containing their various Uſes in

ARITHMETIC, GEOMETRY, TRIGONOMETRY, ARCHITECTURE, SURVEYING, GUNNERY, &c.

With a ſhort Account

Of the AUTHORS who have treated on the

PROPORTIONAL COMPASSES

And SECTOR.

To which is now added

An APPENDIX;

Containing, the Deſcription and Uſe of the

GUNNERS CALLIPERS.

The ſecond Edition, with many Additions.

By J. ROBERTSON, F. R. S.
Maſter of the ROYAL-ACADEMY at *Portſmouth*.

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ERRATA.

Page xvi. read March 5, 1757. p. 10. Line 16. for Plate iii read Plate iv. p. 12. l. 15. for KF r. KC. p. 26. l. 26. for into r. in. p. 40. l. 19. for Scale r. Scales. p. 63. l. 6. dele K. l. 29. for B3 r. B5. p. 80. l. 9. r. As the divisor, is to unity; so is the dividend, to the quotient. And as the divisor, is to the dividend; so is unity, to the quotient. p. 81. l. 19. for 25 r. 35. p. 95. l. 23. for xvii r. xviii. p. 96. l. 12. read Ex. 1. Pl. vi. Fig. 26. p. 100. l. 4. for N. r. x. l. 20. for N. r. x. p. 101. l. 34. for N r. x. l. 35. for M r. z. l. 36. for N r. x. p. 107. l. 1. for xviii r. xix. p. 109. l. 17. for on B read on P. p. 110. In the computation for the letter C read D. p. 125. l. 24. for xix read xx.

Page 128. p. 12. for $Ac=\frac{1}{n+1}N$ read $Ac=\frac{n}{n+1}N$.

Line 16 for $\frac{n}{1}$ read $\frac{1}{n}$. p. 145. l. 19. for $\frac{2}{5}$ r. $\frac{2}{3}$. p. 149. l. 24. for balks r. bulks. p. 155. l. 19 for 1. 8. r. 1. 0.

TO

PETER DAVALL, Esq;

SECRETARY to the ROYAL SOCIETY.

SIR,

IT is no new thing for a lover of Science to addreſs his productions to a friend eminently diſtinguiſhed for his general knowledge, as well as particular ſkill in the parts whereon the Author writes: On this account I heartily wiſh, that inſtead of the ſubjects contained in the following ſheets, I had a work of a more elevated kind wherewith to do greater honour to the name of my friend; however, ſuch as they are, I hope they will, with your uſual frankneſs and good-nature, be accepted. Indeed I muſt obſerve, that the late Preſident of the Royal Society, MARTIN FOLKES, Eſq; honoured the firſt Edition of this book with his Patronage; and alſo, our much-eſteemed and learned friend JAMES BURROW, Eſq; Vice-preſident of the Royal Society, thought the book ſo worthy his peruſal, as to remark all the typographical and other errors, and to make ſome uſeful obſervations, a liſt whereof he favoured me with, and for which I truſt you will permit me to take this opportunity of publickly thanking him: Although I am conſcious, that

that you have the higheſt regard for the two reſpectable names, which I here mention out of gratitude; yet I would not be underſtood that you are to accept hereof in this public manner, merely becauſe thoſe conſiderable perſonages have already favoured the Work; I offer this as a tribute for your acquaintance and friendſhip, and flatter myſelf that you will find in this impreſſion ſome things, which if they have not difficulty to recommend them, have at leaſt, I apprehend, ſo much utility accompanying them, as to render the whole in ſome degree intereſting, and perhaps not unworthy the notice of the moſt ſkilful in the Mathematical Sciences. I am,

SIR,

Your moſt obedient

Humble Servant,

J. ROBERTSON.

TO THE

READER.

IT is needleſs to enumerate the many purpoſes, to which *mathematical inſtruments* ſerve; their uſe ſeems quite neceſſary to *perſons* employed in moſt of the active ſtations in life.

The *Architect*, whether *civil*, *military*, or *naval*, never offers to effect any undertaking, before he has firſt made uſe of his *rule* and *compaſſes*; and fixed upon a ſcheme or drawing, which unavoidably requires thoſe *inſtruments*, and others equally neceſſary.

The *Engineer*, cannot well attempt to put in execution any deſign, whether for *defence*, *offence*, *ornament*, *pleaſure*, &c. without firſt laying before his view, the plan of the whole; which is not to be conveniently performed, but by *rulers*, *compaſſes*, &c.

There are indeed, very few, if any good *Artificers*, who have not in ſome meaſure, occaſion for the uſe of one or more *mathematical inſtruments*; and whenever there is required, an accurate drawing of a thing to be executed, or repreſented; that collection of inſtruments, uſually put in *portable caſes*, is then abſolutely neceſſary: And of theſe, the moſt common ones, or others applicable to like ſervice, muſt have been in uſe, ever ſince mankind have had occaſion to provide for the neceſſary conveniencies of life: But the *parallel ruler*, the *proportional compaſſes*, and the *ſector*, are not of any great antiquity.

However, by means of the opportunity, which the author had of conſulting moſt, if not all the principal

pieces, that have been wrote on this ſubject †; he thinks it will ſufficiently appear from what follows, who were the inventors of theſe latter inſtruments; and when they were firſt known and made uſe of.

I. *Gaſpar Mordente*, in his book *on the compaſſes*, printed in folio at *Antwerp*, 1584; gives the conſtruction and uſe of an inſtrument, invented by his brother *Fabricius Mordente*, in 1554; and by him preſented to the emperor *Maximilian* II. in 1572: *Fabricius* preſented it afterwards, with ſome improvements, to *Rodolphus* II. the ſon of *Maximilian*: In 1578, *Gaſpar* ſtudied to apply the inſtrument to various uſes by the command of the then governor of the *Netherlands*. The inſtrument conſiſts of two flat legs, moveable round a joint like a common pair of compaſſes; but the ends or points are turned down at right angles to the legs, ſo as to meet in one point when the legs are cloſed. In each leg there is a groove, with a ſlider fitted to it, carrying a perpendicular point; ſo that theſe alſo appear like one point when the legs are cloſed, and the ſliders are oppoſite. This compaſs is jointly uſed with a rod, containing a ſcale of equal parts; whereof 30 are equal to the length of each leg. As the operations with this compaſs, depend on the properties of ſimilar triangles, therefore its principles are the ſame with thoſe of the ſector: And moſt, or all the problems that are performed by the line of lines only, can with almoſt the ſame eaſe, be performed by theſe; the tranſition from this inſtrument to the ſector is very natural and eaſy.

The uſe of this inſtrument, is exemplified in problems concerning lines, ſuperficies, ſolids, and meaſuring of inacceſſible diſtances.

The author, p. 22, ſays, he invented an inſtrument there deſcribed; which is our parallel ruler with parallel bars: The parallel ruler with croſs bars, is a more modern contrivance.

† In the collection of the late *William Jones*, Eſq;

II.

II. *Daniel Speckle*, in the year 1589, publiſhed in folio, his *military architecture*, at *Straſburg*; where he was architect. In his ſecond chapter, he takes notice of compaſſes then in uſe of a curious invention, whoſe center could be moved forwards or backwards, ſo that by the figures and diviſions mark'd thereon, a right line could be readily and correctly divided into any number of equal parts, not exceeding 20. This inſtrument has been ſince called the *proportional compaſſes*.

In the ſame chapter he mentions another compaſſes, with an immoveable center, and broad legs, whereon were drawn lines proceeding from the center, and divided into equal parts; whereby a right line could be divided into equal parts not exceeding 20; becauſe the diviſions on the lines ſtill kept the ſame proportion, to whatever diſtance the legs were opened. This inſtrument was afterwards call'd the *ſector*.

III. Dr. *Thomas Hood*, printed at *London*, *Anno* 1598, a quarto book, intituled, *The making and uſe of a Geometrical Inſtrument called a Sector*. This inſtrument conſiſts of two flat legs, moveable about a joint; on theſe are ſectoral lines, of equal parts, of polygons, and of ſuperficies; that is, lines ſo diſpoſed, as to make all the operations that depend on ſimilar triangles quite eaſy, and that without the laying down of any figure. To the legs is fitted a circular arc, an index moveable on a joint, and ſights, whereby it is made fit to take angles.

IV. *Chriſtopher Clavius*, in his *practical geometry*, printed in quarto at *Rome*, *Anno* 1604, in page 4, ſhews the conſtruction and uſe of an inſtrument, which he calls the *inſtrument of parts*; it conſiſts of two flat rulers moveable on a joint; on one ſide of theſe legs, are the ſectoral lines of equal parts; on the other ſide, are thoſe of the chords: After ſhewing ſome of their uſes, he concludes with ſaying, he is ſenſible of many others to which it may be applied,

but leaves them for the exercise of the reader to discover.

V. *Levinus Hulsius*, in his book of *mechanical instruments*, printed in quarto at *Frankfort*, *Anno* 1605; gives, in the third part, the description and use of an instrument, which *Justus Burgius* call'd the *proportional Compass*. *Hulsius* says, the use of it had not been published before, although the instrument had been long known.

VI. *Anno* 1605, *Philip Horscher*, M. D. published at *Mentz*, a quarto book, containing the use and construction of the *proportional compasses*. This author does not pretend to be the inventor; but that seeing such an instrument, he thought he could, from *Euclid*, shew its construction and the grounds of its operations.

VII. *Anno* 1606, *Galilæus* published in *Italian*, a treatise of the use of an instrument which he calls, *The geometrical and military compass*. On this instrument are described sectoral lines of equal parts, surfaces, solids, metals, inscribed polygons, polygons of given areas, and segments of circles. In the preface to an edition of this book, printed at *Padua*, *Anno* 1640, by *Paola Frambotti*, *Galilæus* says, that on account of the opportunity he had of teaching mathematics at *Padua*, he thought it proper to seek out a method of shortening those studies. In another part of the preface he says, that he should not have published this tract, but in vindication of his own reputation; for he was informed that a person had by some means or other, got one of his instruments, and pretended to be the inventor, although himself had taught it ever since the year 1597.

VIII. *Anno* 1607, *Baldessar Capra*, published a treatise of the construction and use of the *compass of proportion*, (or sector.) He claims the invention of this instrument; and hence arose a dispute between *Galilæus* and *Capra*; some particulars of which have been

been mentioned by ſeveral, and particularly by *Thomas Saluſbury*, Eſq; in his life of *Galilæus*, publiſhed at the end of the ſecond volume of his *mathematical collections and tranſlations*, at *London*, in fol. *Anno* 1664.

It appears from theſe accounts that one *Simon Marius* a German who was in *Padua* about the year 1607, tranſlated into latin, the book publiſhed the year before by *Galilæus*, and cauſed his diſciple *Capra* to print it as his own: *Marius* dreading a proſecution, retired, and left *Capra* in the lurch, who was proceeded againſt. At that time *Galilæus* publiſhed an apology, intitled, " *The defence of* Galilæus Galilæi, *a* Florentine *gentleman, reader in the univerſity of* Padoua, *againſt the calumnies and impoſtures of* Baldeſſar Capra *a* Milaneſe, *divulged againſt him as well in his conſideratione aſtronomica upon the new ſtar of* 1604, *as (and more notoriouſly) in lately publiſhing for his invention the conſtruction and uſes of the geometrical and military compaſs, under the title of Uſus & Fabrica circini cujuſdam proportionis, &c.*" *Galilæus* begins with an addreſs to the reader, wherein he concludes, that a perſon robbed of his inventions, ſuffers the greateſt loſs that can be ſuſtained, becauſe it deſpoileth him of *honour, fame and deſerved glory:*" He proceeds, and ſays, " *into this ultimate of miſeries and unhappineſs of condition*, Baldeſſar Capra, *a* Milaneſe, *with unheard of fraud, and unparallel'd impudence hath endeavoured to reduce me, by lately publiſhing, and committing to the preſs my geometrical and military compaſs, as his proper invention, and as a production of his own wit, (for ſo he calls it in the work itſelf) when it was I alone, that ten years ſince (viz. Anno* 1597) *thought of, found and compleated the ſame, ſo as that no one elſe hath any ſhare in it; and I alone from that time forward imparted, diſcovered and preſented it unto many great princes, and other noble lords; and in fine, only that I a year ſince cauſed the operations thereof to be printed, and conſecrated to the glorious name of the moſt ſerene prince of* Tuſcany, *my*

 lord.

lord. Of which said instrument the above-named Capra, *hath not only made himself the author, but reports me for its shameless usurper, (these are his very words) and consequently bound to blush within my self with extream confusion, as unworthy to appear in sight of learned and ingenuous men.*" *Galilæus* then proceeds, among other things, to produce the attestations of four considerable persons, shewing that ten years before that time, he had taught the use of the instrument, and that *Capra* who had for four years past seen them making at the workman's house, had never challenged the invention, as his own.

Galilæus after this, says that he went to *Venice*, and laid the affair before the lords reformers of the university of *Padoua*, on the 8th of *April* 1607, at the same time shewing them his own book, published *June* the 10th 1606; and that of *Capra*'s, published *March* the 7th 1607. The lords thereupon cited *Capra* to appear before them on the 18th of *April*; the next day the cause was heard and the parties dismissed: But on the 4th of *May* following, their lordships pronounced sentence, and sent it to *Padoua* to be put in execution; the amount of their sentence was, that having fully considered the affair, it appeared to them that *Galilæus* had been abused, and that all the remaining copies of *Capra*'s book should be "*brought before their lordships to be suppressed in such fashion as they shall think fit, reserving to themselves to proceed against the printer and bookseller, for the transgressions they may have committed against the laws of printing; ordering the same to be made known accordingly.*

The same day all the copies of the said book were sent to Venice *unto the lords reformers; there being found* 440 *with the bookseller, and* 13 *with the author, he having distributed* 30 *of them into sundry parts of Europe*, &c."

IX. *Anno*

IX. *Anno* 1610, *John Remmelin*, M. D. publiſhed at *Frankfort*, a quarto edition of two tracts of *John Faulhaber*; one of theſe contains the uſe of the *ſector*, on which are lines of equal parts, ſuperficies, ſolids, metals, chords, *&c.* He ſays, that *G. Brendel*, a painter, uſed this inſtrument in perſpective painting.

X. *D. Henrion*, in his *mathematical memoirs*, *Anno* 1612, gave a ſhort tract of the uſe of the compaſs of *proportion* (or *ſector.*) In 1616 he printed a book of the uſe of the ſector; and a fifth edition, in the year 1637, the preface to which, ſeems to be wrote in the year 1626, wherein he ſays, that about the year 1608, he had ſeen in the hands of *M. Alleaume*, engineer to the king of *France*, one of theſe ſectors; whereupon he wrote ſome uſes of it, which he publiſhed in his memoirs, as above. He alſo declares, that before his firſt publication, he had not ſeen any book on the uſe of a ſector, and therefore calls what he publiſhes his own. He charges Mr. *Gunter* with having uſed many of his propoſitions. This author printed at *Paris* 1626, an octavo book of logarithms, at the end of which is a tract call'd logocanon, or the proportional ruler; which is a deſcription and uſe of an inſtrument, he calls a lattice, (perhaps from the chequer-work made by lines drawn thereon) which operates the problems performed by the *french* ſectors very accurately.

XI. *Anno* 1615, *Stephen Michael-Spackers*, publiſhed in quarto at *Ulm*, a treatiſe of the *proportional rule and compaſs of G. Galgemeyer*, reviſed by *G. Brendel*, a painter at *Laugingen*. On theſe proportional compaſſes, are lines of equal parts, of polygons, ſuperficies, ſolids, ratio of the diameter to the circumference; reduction of planes, and reduction of ſolids. The uſe and conſtruction of theſe lines, are ſhewn by a great variety of examples.

XII. *Benjamin Bramer*, in his book of *the deſcription of the propportional ruler and parallelogram*, printed

in

in quarto at *Marpurg*, *Anno* 1617; ſays, his ruler is applicable to the ſame uſes as *Juſtus Burgius*'s inſtrument. *Bramer*'s inſtrument conſiſts of a ruler, on which are lines of equal parts, of ſuperficies, of ſolids, of regular ſolids, of circles, of chords, and of equal polygons; at the beginning of each ſcale, is a pin-hole, whereby he can apply the edge of another ruler, and ſo conſtitute a ſector for each ſcale.

XIII. *Anno* 1623, *Adriano Metio Alcmariano*, printed at *Amſterdam* a quarto book, ſhewing the uſe of an inſtrument called the *rule of proportion*. In his dedication, he ſays, that whilſt he was reviewing ſome things relating to practical geometry, he met with *Galileo*'s book of the uſe of the ſector, which gave him opportunity to improve on it, and occaſioned the publiſhing of this book.

XIV. Mr. *Edmund Gunter*, profeſſor of aſtronomy in *Greſham* college, printed at *London*, *Anno* 1624, a quarto book, called *the deſcription and uſe of the ſector*; on which are ſectoral lines, 1ſt. of equal parts; 2d. ſuperficies; 3d. ſolids; 4th. ſines and chords; 5th. tangents; 6th. rhumbs; 7th. ſecants: Alſo lateral lines of, 8th. quadratures; 9th. ſegments; 10th. inſcribed bodies; 11th. equated bodies; 12th. metals: On the edges are a line of inches and a line of tangents.

Mr. *Gunter* does not ſay any thing concerning the invention, and has no preface; but at the end of the tract, in a concluſion to the reader, he ſays, that the ſector was thus contrived, moſt part of the book written, and many copies diſperſed, more than ſixteen years before, *&c.* this article being written *May* 1, 1623, brings the time he ſpeaks of to about the year 1607, which was before the time *Henrion* ſays he firſt ſaw the ſector.

The ſcales of logarithm numbers, ſines, and tangents, were firſt publiſhed in 1624, in *Gunter*'s deſcription of the croſs ſtaff.

XV. *Mutio Oddi* of *Urbino* printed at *Milan*, *An.*

1633, a quarto book, called *the construction and use of the compasso polimetro*, (or sector.) The lines on this instrument, were such as were common at that time: He says in the dedication to his friend *Peter Linder* of *Nurenberg*, he first taught the use of it

In the preface he says, that about the year 1568, *Commandine*, who then taught at *Urbino*, did contrive a pair of compasses with a moveable centre, to divide right lines into equal parts; which was done at the request of a gentleman named *Bartholomew Eustachio*, who wished to avoid the trouble of the common methods, or of being obliged to have many compasses for such divisions of right lines.

He farther says, that about that time, *Guidibaldo*, marquess of *Monte*, who lived at *Urbino* for the sake of *Commandine*'s company, being frequently at the house of *Simone Boraccio*, who made *Commandine*'s proportional compasses, did contrive, and cause to be made, an instrument with flat legs, (like the sector) which performed the operations of the compass more easily. *Oddi* says also, that great numbers were made, and in few years, had many useful and curious additions, with treatises written on its use in diverse languages, and called by different names, which occasioned the doubt of who was the true author, every one having found means to support his cause: But *Oddi* says, he not intending to decide the dispute, leaves it to time to discover; and seems contented to have pointed out who was the first inventor; his chief intention being that of making the use public, and the construction easy to workmen.

The following authors have also wrote on the sector, and sectoral lines.

XVI. *Anno* 1634, *P. Petit*, printed in 8vo. at *Paris*, a treatise on the sector. He thinks *Galilæus* was the inventor.

XVII. *An.* 1635, *Matthias Berneggerus* printed at *Strasburg* a 4to. edition of *Galilæus*'s book on the sector, which consists of two parts: To this is added a third

a third part, shewing the construction of *Galilæus*'s lines, and some additional uses and tables.

XVIII. *An.* 1639, *Nicholas Forest Duchesne* printed at *Paris*, in 12mo. a book of the sector. He seems to be little more than a copier of *Henrion*.

XIX. *An.* 1645, *Bettinus* in his *Apiaria universa*, &c. *apiar.* 3d. p. 95, and *apiar.* 12, p. 4. In his *Ærarium philo. math.* 4to. *an.* 1648, vol. I. p. 262. In his *Recreationum math. appiariæ*, &c. 12mo. *an.* 1658, p. 75, applies the sector to music.

XX. *John Chatfield* printed at *London*, in 12mo. his *trigonal sector*, *anno* 1650.

XXI. *An.* 1656, *Nicholas Goldman* printed at *Leyden*, in folio, his *treatise on the sector*. He says that *Galilæus* was the first who published the description of the sector, an invention useful in all parts of the mathematics, and other affairs of life.

XXII. *John Collins* printed at *London*, in 4to. his book of *the sector on a quadrant*, *an.* 1659.

XXIII. *Pietro Ruggiero*, in his *military architecture*, in 4to. printed at *Milan*, *an.* 1661, p. 230, applies the sector to the practice of fortification.

XXIV. *An.* 1662, *Gaspar Schottus* printed at *Strasburgh* his *mathesis cæsaræa*, in 4to. in which he gives a description and use of the sector: In the preface he mentions *Galilæo* as the inventor of the sector.

XXV. *J. Templar* printed in 12mo. at *London*, *an.* 1667, a book called *the semicircle on a sector*. He says, the applying of Mr. *Forster*'s line of versed sines to the sector, was first published *an.* 1660, by *John Brown*, mathematical instrument-maker in *London*.

XXVI. *Daniel Schwenter* in his *practical geometry*, revised and augmented by *George Andrew Bocklern*, printed in 4to. at *Nuremberg*, *an.* 1667, treats on the description and use of the sector.

XXVII. *John Caramuel* printed at *Campania*, *an.* 1670, his *mathesis nova*, in 2 vols. folio. In the 2d vol.

vol. p. 1158, he treats on the ſector, relates the conteſt between *Galilæus* and *Capra*, and thinks the ſame might have been objected againſt others, as well as againſt *Capra:* He alſo ſays, that *Clavius* had ſuch an inſtrument before that of *Galilæus* appeared; and *Clavius* having taught for a long time at *Rome*, had many ſcholars, ſome of whom might have carried his inſtruments to ſeveral countries. *Caramuel* mentions a ſtory of a *Hollander* ſhewing to *Galilæus* an inſtrument of this ſort, that he had brought from his country, and of which *Galilæus* took a copy.

XXVIII. *John Brown*, in his book on *the triangular quadrant*, printed in 8vo. at *London*, *an.* 1671.

XXIX. *John Christopher Rohlhans*, in his *math. and optical curioſities*, printed in 4to. at *Leipſic*, *an.* 1677, p. 216.

XXX. *An.* 1683, *Staniſlawa Solſkiego* printed at *Kracow*, his *geometria et architectura Polſki*, in folio. p. 69, treats on ſome ſectoral lines.

XXXI. *Henrick Jaſper Nuis*, printed at *Tezwolle*, in 4to. his *Rectangulum catholicum geometrico aſtronomicum*, *an.* 1686.

XXXII. *De Chales*, in his *curſus mathem.* printed at *Leyden*, in 2 vols. fol. *an.* 1690. Vol. 2d. p. 58, relates the conteſt between *Galilæus* and *Capra*, and aſcribes the invention of the proportional compaſs to Dr. *Horſcher*, or *Juſtus Burgius*.

XXXIII. *An.* 1691, an edition in 8vo. of Mr. *Ozanam*'s treatiſe of the ſector, was printed at the *Hague*.

XXXIV. *P. Hoſte* printed at *Paris* his courſe of mathematics, in 3 vols. 8vo. *an.* 1692. In vol. 2d. p. 27. he gives a tract on the ſector.

XXXV. *Thomas Allingham* in his *ſhort treatiſe on the ſector*, in 4to. *London*, 1698.

XXXVI. *J. Good*, in his *treatiſe on the ſector*, in 12mo. *London*, 1713.

XXXVII. *Christian Wolfius*, in his *math*, *lexicon*, 8vo. printed at *Leipsic*, *an.* 1716, under the word *circinus proportionum*, relates, that *Levinus Hulsius*, in his treatise on the proportional compasses, printed at *Frankfort* the 10th of *May*, 1603, says, that he first saw the said instrument at *Ratisbon*, on the day of the imperial dyet: That he had sold them far and near before 1603; and that it had been inaccurately copied in several places: *Wolfius* says farther, that *Justus Burgius* was certainly the inventor, but used to let his inventions lye unpublished.

He then relates the contest between *Galilæus* and *Capra*, and ends with shewing the difference between the instruments of *Burgius* and *Galilæus*.

XXXVIII. *M. Bion*, in his construction of mathematical instruments, translated by *Edmund Stone*, fol. *London*, 1723.

XXXIX. Mr. *Belidor*, in his *new course of math.* in 4to. p. 364, *Paris*, 1725.

XL. *Roger Rea*, in his *sector and plane scale compared*, 8vo. *London*, 1727, 2d edition.

XLI. *Vincent Tosco*, in his *compendium of the math.* in 9 vols. 8vo. *Madrid*, 1727, vol. I. p. 359.

XLII. *Jacob Leupold*, in his *theatrum arithmetico-geometricum*, in fol. *Leipsic*, 1727. p. 86, gives a detail of the inventors of the proportional compasses and sector, which goes on to p. 121, and then he gives a list of the authors who have wrote on proportional instruments, *viz. Bramer*, 1617; *Capra*, 1607; *Casati*, 1664; *Conette*, 1626; *Dechales*, 1690; *Dolz*, 1618; *Faulhaber*, 1610; *Galgemeyer*, 1615; *Brendell*, 1611: *Galilæus*, 1612; *Goldman*, 1656; *Horscher*, 1605; *Horen*, 1605; *Hulsius*, 1604; *Clavius*, 1615; *Lockmann*, 1626; *Metius*, 1623; *Patridge*—; *de Saxonica*, 1619; *Scheffelts*, 1697; *Steymann*, 1624; *Uitenhoffers*, 1626.

XLIII. *Samuel Cunn*, in his *new treatise on the sector*, 8vo. *London*, 1729.

XLIV.

XLIV. *William Webſter*, in his appendix to a tranſlation of *P. Hoſt*'s mathematics, 8vo. 2 vols. *London*, 1730.

There may be ſeveral other authors who have wrote on the conſtruction and uſe of the ſector, or on ſome of the ſectoral lines; but thoſe above, are all that have come to hand; and indeed theſe are many more than are wanted to determine this enquiry; which may be collected chiefly, from *Mordente*, *Speckle*, *Hood*, *Clavius*, *Hulſius*, *Galilæus*, *Oddi*, *Salluſbury*, *Caramuel*, *Dechales*, *Wolfius*, and *Leupold*; the others ſerving only to inform the reader what works are extant on this ſubject. From the whole he may obſerve, that there are few countries in *Europe*, but have one or more treatiſes on the proportional compaſſes and ſector, in their own language; and this is ſufficient to ſhew, that theſe inſtruments have been in univerſal eſteem.

As the publication of *Mordente*'s book was in 1584, it is not improbable, as *Caramuel* relates, that a *Hollander* (or one from the neighbourhood of *Antwerp*) might ſhew one of *Mordente*'s inſtruments to *Galilæus*: Neither is it improbable that *Galilæus* had ſeen both *Mordente*'s and *Speckle*'s books, the former having been publiſhed thirteen years, and the latter eight years, before *Galilæus*, by his own accounts, thought of his inſtrument.

As *Mutio Oddi*, was a native of *Urbino*, and from what he ſays in his dedication, it is not improbable but he was acquainted with one or more of the perſons he mentions in his preface, or at leaſt with ſome of their acquaintance, from whom he might gather the particulars he relates; to which, if any credit may be given, *Commandine* was the inventor of the proportional compaſſes, and *Guidobaldo* of the ſector: And in the intercourſe between *Italy* and *Germany*, ſome of *Simone Borachio*'s work might get into the hands of many ingenious *Germans*, and give *Juſtus*

Burgius,

Burgius, to whom the proportional compaſs is uſually aſcribed, opportunity of getting an early copy; and alſo put into *Speckle*'s way, the inſtrument he mentions to have ſeen: His deſcription pretty nearly agreeing with what *Oddi* ſays was contrived by *Guidobaldo*.

But while we are ſearching among foreigners for the inventor of the ſector, what are we to think of our countryman Dr. *Hood?* who in 1598 publiſhed his account of an inſtrument which he really calls a ſector: And though we ſhould allow that *Hood* as well as *Galilæus* might have ſeen *Mordente*'s and *Speckle*'s books; and both of them might have ſeen ſome of *Borrachio*'s work, yet it is not very probable that *Hood* could have got the form of his inſtrument from *Galilæus* the year after he thought of it; and as *Hood* publiſhed eight years before *Galilæus*, *Hood* certainly has an equal right with *Galilæus*, if not a greater, to the honour of the invention of the ſector.

After all, it may be ſaid, that it is not impoſſible for the ſame thing to be diſcovered by different perſons who have no connexion with one another; examples of a like coincidence of thoughts being known on other ſubjects.

To the preſent edition, there is added an appendix on the gunners callipers, which was promiſed to the public in the former impreſſion, publiſhed at the beginning of the year 1747; and beſide this, the body of the book has been augmented by more than three ſheets of additional illuſtrations and problems, and another plate: By all theſe additions, it is conceived the book is now rendered more generally uſeful.

What is done in the foregoing eſſay, and in the following work, is ſubmitted to the reader's judgment; the author intending no more than to have the honour of invention aſcribed to whom it is due; and alſo to give ſome aſſiſtance to beginners in the mathematical ſtudies.

Royal Academy Portſmouth
March 5, 1755.

CON-

THE CONTENTS.

The figures referred to, are contained in seven copper-plates.

APPENDIX.

To the Binder.

The plates are all to stand upright in the book, and no part to be folded upwards or downwards.

THE

DESCRIPTION *and* USE

OF A

C A S E,

OR

PORTABLE COLLECTION,

Of the most Necessary

Mathematical Instruments.

SECT. I.

CASES of *Mathematical Instruments* are of various sorts and sizes; and are commonly adapted to the fancy or occasion of the persons who buy them.

THE smallest collection put into a case, commonly consists of,

I. A *pair of compasses*, one of whose points may be taken off, and its place supplied with,

 A *crayon*

A *crayon* for lead or chalks.

A *drawing-pen* for ink.

II. A *plane scale.*

WITH these instruments only a tolerable shift may be made to draw most mathematical figures.

BUT in sets, called *complete pocket-cases*, beside the instruments above, are the following.

III. A smaller pair of *compasses.*

IV. A pair of *bows.*

V. A *black-lead pencil*, with a *cap* and *feeder.*

VI. A *drawing-pen* with a *protracting-pin.*

VII. A *protractor.*

VIII. A *parallel-ruler.*

IX. A *sector.*

IN some cases, the plane scale, protractor, and parallel-ruler, are included in one instrument.

THE common, and most esteemed size of these instruments, is six inches; though they are sometimes made of other sizes, and particularly of four inches and a half.

Note, the size of a case is named from the length of the scale or sector.

SOME artists have contrived a very commodious flat case, or box, where the inside of the lid or top contains the rulers and scales: The compasses, drawing-pen, &c. lie in the partitions of a drawer, that drops into the bottom part of the case, but not quite to the bottom; leaving room under it for *black lead pencils*, *hair pencils*, *Indian ink*, *colour cells*, &c. and beside the instruments already enumerated, in boxes or cases of this sort are put

X. A *tracing-point.*

XI. A pair of *proportional compasses.*

XII. A *gunner's callipers.*

BUT the case of instruments called the *magazine*, is the most complete collection; for this contains whatever can be of use in the practice of *drawing*, *designing*, &c. and as the greatest part of these instruments

are

are ſcarcely ever uſed but in the ſtudies or chambers of thoſe who have occaſion for them; therefore it will be uſeleſs to inſiſt on pocket caſes; for few perſons care to load themſelves with the carriage of what is called a *complete ſet.*

SECT. II.

Of the COMPASSES *and* BOWS.

COMPASSES are uſually made of ſilver or braſs, and thoſe are reckoned the beſt, part of whoſe joint is ſteel; and where the *pin* or *axle* on which the joint turns, is a ſteel ſcrew; for the oppoſition of the metals makes them wear more equable: and by means of the ſcrew axle, with the help of a *turn-ſcrew*, (which ſhould have a place in the caſe) the compaſſes can be made to move in the joint, ſtiffer or eaſier, at pleaſure. If this motion is not uniformly ſmooth, it renders the inſtrument leſs accurate in uſe. Their points ſhould be of ſteel, and pretty well hardened, elſe in taking meaſures off the ſcales, they will bend, or be ſoon blunted. They alſo ſhould be well poliſhed, whereby they will be preſerved free from ruſt a long time.

To one point of the ſmaller compaſſes, it is common to fix in the ſhank a ſpring, which by means of a ſcrew, moves the point; ſo that when the compaſs is opened nearly to a required diſtance, by the help of the ſcrew the points may be ſet exactly to that diſtance; which cannot be done ſo well by the motion in the joint.

To uſe the ſpring point.

HOLD the compaſſes in the left hand with the ſcrew turned towards the right; turn the ſcrew towards you,

or ſlacken it, and the ſpring point will be brought nearer to the other point: On the contrary, by turning the ſcrew from you, or tightning it, the ſpring point will be ſet farther from the other point.

The uſe of theſe leſſer compaſſes, is to transfer the meaſures of diſtances from one place to another; or, to deſcribe obſcure arcs.

Of the large ſized compaſſes, thoſe are eſteemed the beſt, whoſe moveable points are locked in by a ſpring and catch fixed in the ſhank; for if this ſpring be well effected, the point is thereby kept tight and ſteady; the contrary of which frequently happens, when the point is kept in by a ſcrew in the ſhank.

The uſe of theſe compaſſes is to deſcribe arcs or circumferences with given radius's: and it is eaſy to conceive, that theſe arcs or circumferences can be deſcribed, either obſcurely by the ſteel point; in ink, by the ink point; in black-lead or chalks, by the crayon; and with dots, by the dotting-wheel; for either of them may be fixed in the ſhank in the place of the ſteel point.

As the dotting-wheel has not hitherto been effected, ſo as to deſcribe dotted lines or arcs, with any tolerable degree of accuracy, it ſeems therefore to be uſeleſs: and, indeed, dotted lines of any kind are much better made by the drawing-pen.

The drawing-pen point, and crayon, have generally (in the beſt ſort of caſes) a ſocket fitted to them: ſo that they occupy but one of the holes, or partitions, in the caſe.

The ink, and crayon points, have a joint in them, juſt under that part which locks into the ſhank of the compaſſes; becauſe the part below the joint ſhould ſtand perpendicular to the plane on which the lines are deſcribed, when the compaſs is opened.

If inſtead of the larger compaſs being made with ſhifting points, there were two pair put into the caſe; to one of which the ink point was fixed, and to

the other the crayon point; this would save the trouble of changing the points in the compass at every time they were used; and would increase the expence, or bulk of the case, but a trifle.

Most persons at first, handle a pair of compasses very aukwardly, whether in the taking of distances between the points, or describing of circles. To be sure long practice brings on easy habits in the use of things, however a caution or two may be serviceable to beginners.

To open and work the compasses.

With the thumb and middle finger of the right hand pinch the compasses in the hollow part of the shank, and it will open a little way; then the third finger being applied to the inside of the nearest leg, and the nail of the middle finger acting against the farthest, will open the compasses far enough to introduce the fingers between the legs: then the hither one being held by the thumb and third finger, the farther leg may be moved forwards and backwards very easily by the fore and middle fingers, the fore finger pressing on the outside to shut, and the middle one acting on the inside to open, the compasses to any desired extent. In this manner the compasses are manageable with one hand, which is convenient when the other hand is holding a ruler or other instrument.

To take a distance between the points of the compasses.

Hold the compasses upright, set one point on one end of the distance to be taken, there let it rest; and (as before shewn) extend the other point to the other end.

Always take care to avoid working the compasses with both hands at once; and never use them otherwise than nearly upright.

To describe circles or arcs with the compasses.

Set one foot of the compasses on the point designed for the centre, hold the head between the thumb and middle finger, and let the fore finger rest on the head, but not to press it: then by rolling the head between the finger and thumb, and at the same time touching the paper with the other point, a circle or arc may be described with great ease, either in lead or ink.

In describing of arcs it should be observed, that the paper be not pressed at the centre, or under the foot, with more weight than that of the compasses; for thereby the great holes and blots may be avoided, which too frequently deface figures when they are made by those who are aukward or careless in the use of their instruments.

Of the Bows.

The bows are a small sort of compasses, that commonly shut into a hoop, which serves as a handle to them. Their use is to describe arcs, or the circumferences of circles, whose radius's are very small, and could not be done near so well by larger compasses.

Sect. III.

Of the Black-lead Pencil, Feeder, and Tracing Point.

THE *Black-lead Pencil* is useful to describe the first draught of a drawing, before it is marked with ink; because any false strokes, or superfluous lines,

lines, may be rubb'd out with a handkerchief or piece of bread.

THE *Feeder* is a thin flat piece of metal, and is sometimes fixed to a cap that slips on the top of the pencil, and serves either to put ink between the blades of the drawing-pen, or to pass it between the points, when the ink by drying, does not flow freely.

THE *Tracing Point* is a pointed piece of steel; and commonly has the feeder fixed to the other end of the handle. Its use, is to mark out the outlines of a drawing or print when an exact copy thereof is wanted, which may be done as follows.

ON a piece of paper, large enough to cover the thing to be copied, let there be strewn the scrapings of *red chalk*, or of *black chalk*, or of *black lead*; rub these on the paper, so that it be uniformly covered; and wipe off, with a piece of muslin, as much as will come away with gentle rubbing. Lay the coloured side of this paper, next to the vellum, paper, *&c.* on which the drawing is to be made: on the back of the colour'd paper, lay the drawing, *&c.* to be copied. Secure all the corners with weights, or pins, that the papers may not slip: trace the lines of the thing to be copied, with the *tracing point*; and the lines so traced will be impress'd on the clean paper.

AND thus, with care, may a drawing or print, be copied without being much damaged.

Note, The coloured paper will serve a great many times.

THERE is not perhaps, a more useful instrument in being for ready service in making of sketches or finished plans; whether of architecture, fortification, machines, landskips, ornaments, *&c.* than a black-lead pencil; and therefore it may be proper to give a few hints concerning this excellent mineral.

BLACK-LEAD is produced in many countries, but the best yet discovered is found in the north of England: it is dug out of the ground in lumps, and sawed out

into ſcantlings proper for uſe: the kinds moſt proper to uſe on paper muſt be of an uniform texture, which is diſcoverable by paring a piece to a point with a penknife; for if it cuts ſmooth and free from hard flinty particles, and will bear a fine point, it may be pronounced good.

THERE are three ſorts of good black-lead; the ſoft, the midling, and the hard: the ſoft is fitteſt for taking of rough ſketches, the midling for drawing of landſkip and ornaments, and the hard for drawing of lines in mathematical figures, fortification, architecture, *&c.* The indifferent kinds, or thoſe which in cutting are found flinty, are uſeful enough to carpenters or ſuch artificers who draw lines on wood, &c.

THE beſt way of fitting black-lead for uſe, is firſt to ſaw it into long ſlips about the ſize of a crow-quill, and then fix it in a caſe of ſoft wood, generally cedar, of about the ſize of a gooſe-quill, or larger; and this caſe is cut away with the lead as it is uſed.

SECT. IV.

Of the Drawing-Pen, and Protracting-Pin.

THE *Drawing-pen* is an inſtrument uſed only for drawing of right lines; and conſiſts of two blades, with ſteel points, fix'd to a handle. The blades by being a little bent, cauſe the ſteel points to come nearly together; but by means of a ſcrew paſſing thro' both of them, they are brought cloſer at pleaſure, as the line to be drawn ſhould be ſtronger or finer.

IN uſing this inſtrument, put the ink between the blades with a common *pen*, or with the *feeder*; and (by the ſcrew) bring them to a proper diſtance for drawing the intended line: hold the *pen* a little inclined,

clined, but so that both blades touch the paper; then may a line be drawn very smooth, and of equal breadth, which could not be done so well with a common *pen.*

Note; BEFORE the *drawing-pen* is put into the case, the ink should be wiped from between the blades; otherwise they will soon rust and spoil, especially with common ink. And that they may be clean'd easily, one of the blades should move on a joint.

THE directions given about this *drawing-pen*, will serve for the *drawing-pen* point, used with the compasses. But it must be observed, that when any arc is described of more than an inch radius, then the ink point should be bent in the joint so that both the blades of the pen touch the paper, otherwise the arc described will not be smooth.

THE *Protracting-pin* is a piece of pointed steel (like the point of a needle) fixed into one end of a part of the handle of the *drawing-pen*; into which, the piece with the *pin* in it, generally screws. Its use is to point out the intersections of lines; and to mark off the divisions of the protractor, as hereafter directed.

SOMETIMES on the top of the *drawing-pen* is a socket, into which a piece of black-lead pencil may be put.

SECT. V.

Of the PARALLEL-RULER.

THIS instrument consists of two *Rulers*, connected together by two metal bars, moving easily round the rivets which fasten their ends; these bars are so placed that both have the same inclination to

to each *Ruler*; whereby they will be *Parallel* at every diſtance, to which the bars will ſuffer them to receed.

But the beſt *Parallel-Rulers* are thoſe, whoſe bars croſs each other, and turn on a joint at their interſection; one end of each bar moving on a centre, and the other ends ſliding in grooves as the *Rulers* receed.

This inſtrument is very uſeful in delineating civil and military architecture, where there are many *Parallel* lines to be drawn; and alſo in the ſolution of ſeveral geometrical *Problems*; ſome of which are as follows.

PROBLEM I.

A right line AB *being given, to draw a line parallel thereto, that ſhall paſs through a given point* C (Fig. 1. Pl. ~~III~~) IV.

Construction. Apply one edge of the *parallel-ruler* to the given line AB; preſs one *ruler* tight againſt the paper, and move the other untill its edge cuts the point C; there ſtay that *ruler*, and by its edge draw a line through C, then this line will be *parallel* to AB.

If the point C happens to be farther from the line AB, than the *rulers* will open to; ſtay that *ruler* neareſt to C, and bring the other cloſe to it, where let it reſt, and move forward the *ruler* neareſt to C, and ſo continue till one *ruler* is brought to the point intended.

The manner of uſing the *parallel-ruler* as here directed, is underſtood to be the ſame in the ſolution of the following Problems.

PROBLEM II.

A right line AB *being given, to divide it into any propos'd number of equal parts; ſuppoſe* 5. (Fig. 2.)

Construction. Draw the indefinite right line BC, ſo as to make with AB, any angle at pleaſure; with any

any convenient opening of the compasses, lay off on BC, the required number of equal parts, *viz.* 1, 2, 3, 4, 5; lay the edge of the *parallel-ruler* by the points 5 and A, and *parallel* thereto, through the points 4, 3, 2, 1, draw lines; then AB, by the intersection of those lines will be divided into 5 equal parts.

PROBLEM III.

Any right lined quadrangle or polygon being given, to make a right lin'd triangle of equal area.

EXAM. I. *To make a triangle of equal area to the quadrilateral* ABDC. (Fig. 3.)

CONSTRUCTION. Prolong AB; draw CB; and through D, draw DE *parallel* to CD, cutting AE in E; then a line drawn from C to E forms the triangle ACE, of equal area to the quadrangle ABDC.

EXAM. II. *Given the pentagon* ABCDE; *requir'd to make a triangle of equal area.* (Fig. 4.)

CONSTRUCTION. Produce DC towards F; draw AC; through B, and *parallel* to AC draw BF cutting DC in F; and draw AF. Then the area of the trapezium AFDE will be equal to the area of the *pentagon* ABCDE.

Again. Produce ED towards G; draw AD; through F, draw FG *parallel* to AD, and draw AG. Then the area of the triangle AGE, will be equal to that of the trapezium AFDE; and consequently, to that of the pentagon ABCDE.

EXAM. III. *To make a triangle equal in area to the Hexagon,* ABCDEF. (Fig. 5.)

CONSTRUCTION. Draw FD, and *parallel* thereto, through E, draw EG meeting CD produced in G, and draw GF. Then the triangle FGD is equal to the triangle FED, and the given *Hexagon* is reduced to the *Pentagon* ABCGF equal in area.

Again.

Again. Draw AG; through F, draw FH *parallel* to AG, meeting CG produced in H; draw AH, and the *pentagon* is reduced to the trapezium ABCH.

Lastly, Draw AC, and *parallel* thereto, through H, draw HI, meeting BC produced in I, and draw AI. Then the trapezium is reduced to the triangle ABI, which is equal in area to the given *Hexagon* ABCDEF.

EXAM. IV. *Given the nine sided figure* ABCDEFGHI, *to make a triangle of equal area.* (Fig. 6.)

CONSTRUCTION. 1st, Draw IB, and through A draw AK *parallel* to IB, meeting HI produced in K, and draw BK; so the three sides HI, IA, AB, are reduced to the two sides HK, KB.

2d, Draw KC, and through B draw BL *parallel* to KF, meeting CD in L; draw KL, and the three sides DC, CB, BK, are reduced to the two sides DL, LK.

3d, Draw KG; through H, draw HM, *parallel* to KG, meeting GF in M, and draw KM; so the three sides KH, HG, GF, are reduced to the sides KM, and MF.

4th, Draw KF; through M, draw MN, *parallel* to KE, meeting FE in N, and draw KN; so the three sides KM, MF, FE, are reduced to two sides KN, NE.

5th, Draw LN, and through K, draw KO, *parallel* to LN, meeting EF produced in O, and draw LO; so the three sides EN, NK, KL, are reduced to the two sides EO, OL.

Lastly, Draw LE, and through D, draw DP *parallel* to LE, meeting OE produced in P, and draw LP; so shall the triangle OLP be equal in area to the given nine sided figure.

PROCEEDING in the same manner; a figure of any number of sides may be reduced to a triangle of equal area.

SECT.

Sect. VI.

Of the Protractor.

THE *Protractor*, is an instrument of a semicircular form; being terminated by a right line representing the diameter of a circle, and a curve line of half the circumference of the same circle. As at Fig. 7. The point c, (the middle of AB) is the centre of the semicircumference ADB, which semicircumference is divided into 180 equal parts call'd degrees; and for the convenience of reckoning both ways, is numbered from the left hand towards the right, and from the right hand towards the left, with 10, 20, 30, 40, &c. to 180, being the half of 360, the degrees in a whole circumference. The use of this instrument is to *protract*, or lay down an angle of any number of degrees, and to find the number of degrees contained in any given angle.

But this instrument is made much more commodious, by transferring the divisions on the semicircumference, to the edge of a *ruler*, whose side EF is *parallel* to AB; (see Fig. 7.) which is done by laying a *ruler* on the centre c, and the several divisions on the semicircumference ADB, and marking the intersections of that *ruler* on the line EF, which may easily be conceiv'd by observing the lines drawn from the centre c to the divisions 90, 60, 30; so that a *ruler* with these divisions mark'd on 3 of its sides and numbered both ways, as in the *Protractor*, (the fourth or blank side representing the diameter of the circle) is of the same use as a *Protractor*, and is much better adapted to a case.

That side of the instrument on which the divisions are mark'd, is call'd the graduated side, or limb of the instrument, which should be sloped away to an edge,

edge, whereby the divisions on the limb will be much easier pointed off.

PROBLEM IV.

A number of degrees being given; to protract, or lay down an angle whose measure shall be equal thereto. And an angle being protracted, or laid down, to find what number of degrees measures that angle.

Pl. V EXAM. I. *To draw a line from the point* A, *that shall make an angle with the line* AB *of* 48 *deg.* Fig. 8.

APPLY the blank edge of the protractor to the line AB, so that the middle or centre thereof (which is always mark'd) may fall on the point A; then with the protracting-pin, make a mark on the paper against the division on the limb of the instrument numbered with the degrees given; (*viz.* 48.) counting from the right hand towards the left; a line drawn from A, through the said mark, as AC, shall with AB, form the angle required, *viz.* 48 degrees.

IF the line had been to make an angle with AB, at the point B; then the centre must have been laid on B, and the divisions counted from the left hand towards the right.

EXAM. II. *To find the number of degrees which measure the angle* ABC. Fig. 9.

APPLY the blank edge of the protractor to the line AB, so that the centre shall fall on the point B; then will the line BC cut the limb of the instrument in the number expressing the degrees that measure the given angle; which in this *example* is 125 degrees, counting from the left hand towards the right.

PROBLEM V.

From any given point A, *in a line* AB, *to draw a line perpendicular to* AB. Fig. 10.

Lay the protractor acroſs the line AB in ſuch a manner that the centre on the blank edge, and the diviſion numbered with 90, on the limb, may both be cut by the given line; then keeping the ruler in this poſition, ſlide it along the line, till one of theſe points touch the given point A, draw the line CA, and it will be perpendicular to AB.

In the ſame manner, a line may be drawn, perpendicular to a given line, from a given point out of that line.

PROBLEM VI.

In a circle given to inſcribe any regular Polygon. Suppoſe an octagon. Fig. 11.

Construction. Apply the blank edge of the *protractor* to AB the diameter of the *Circle*, ſo that their centres ſhall coincide; ſet off a number of degrees from B to D equal to an angle at the centre of that *polygon*, (viz. 45.) and through that mark draw a radius CD; then ſhall BD the chord of the arc expreſſing thoſe degrees, be the ſide of the intended *polygon*; which chord taken between the compaſſes, and applied to the circumference will divide it into as many equal parts as the *polygon* has ſides, *viz.* 8; and the ſeveral chords being drawn will form the *polygon* required.

It will rarely happen that this operation, though true in theory, will give the ſide of the *polygon* exact; for when the chord of the arc prickt off from the protractor, is taken with the compaſſes and applied to the circle, it generally falls beyond, or ſhort, of the point ſet out from: for it muſt be obſerved that the point where two lines in-

terſect one another is not to be readily determined in a practical manner; and a very ſmall error in the taking the length of the chord, being ſeveral times repeated becomes conſiderable at laſt. Here the compaſſes with the ſpring point will be found of great uſe.

A TABLE, *ſhewing the Angles at the Centres and Circumferences of regular* Polygons *from three to twelve Sides incluſive.*

Names.	Sides.	Angles at Center		Angles at Cir.	
Trigon	3	120°	00′	60°	00′
Square	4	90	00	90	00
Pentagon	5	72	00	108	00
Hexagon	6	60	00	120	00
Heptagon	7	51	$25\frac{5}{7}$	128	$34\frac{2}{7}$
Octagon	8	45	00	135	00
Nonagon	9	40	00	140	00
Decagon	10	36	00	144	00
Endecagon	11	32	$43\frac{7}{11}$	147	$16\frac{4}{11}$
Dodecagon	12	30	00	150	00

THIS table is conſtructed, by dividing 360, the degrees in a circumference, by the number of ſides in each polygon; and the quotients are the angles at the centers; the angle at the center ſubſtracted from 180 degrees, leaves the angle at the circumference.

PROBLEM VII.

Upon a given right line AB, *to deſcribe any regular polygon.* Fig. 12.

CON-

CONSTRUCTION. From the ends of the given line, draw the lines AD, BC; so that the angles BAD, ABC, may each be equal to the angle at the circumference in that polygon; make AD, BC, each equal to AB; from the points D and C, draw lines that shall make with DA, CB, angles equal to the former; make these lines each equal to AB; and so continue, till a polygon is form'd of as many sides as required.

EXAM. I. *Upon the line* AB *to describe an hexagon.* Fig. 12.

DRAW AD, BC, so that the angles BAD, ABC, may be each 120 degrees; make AD, BC, each equal to AB: also, make the angles ADF, BCE, each equal to 120 degrees, and make DF, CE, each equal to AB; draw FE and 'tis done.

OR it may be done by the help of the parallel ruler, when the polygon has an even number of sides, Thus,

HAVING form'd the three sides AD, AB, BC, as before directed; through D, draw DF parallel to BC; make DF equal to AB; through F draw FE parallel to AB: make FE equal to AB and join CE.

EXAM. II. *Upon the line* AB *to describe a pentagon.* Fig. 13.

DRAW AC, BD, that each may make with AB, an angle of 108 degrees. Make AC, BD, each equal to AB; on the points C and D, with the compasses opened to the distance AB, describe arcs to cross each other in E; draw EC and ED, and 'tis done.

IN any regular polygon, having found all the sides but two, as above directed; those may be found as the last two in the pentagon were.

BUT a regular polygon described upon a given line AB may be constructed with more accuracy, thus. See Fig. 12, 13.

Make an angle BAP, and another ABP, each equal to half the angle of the required polygon; on the point P, where the lines AP, BP, cut one another, and with the radius PA describe a circle, in which if the given line AB be applied, the polygon sought will be formed.

SECT. VII.

Of the Plain Scale.

THE lines generally drawn on the plane scale, are these following:

			Marked
I.	Lines of equal parts.		E P.
II.	———	Chords.	Cho.
III.	———	Rhumbs.	Ru.
IV.	———	Sines.	Sin.
V.	———	Tangents.	Tan.
VI.	———	Secants.	Sec.
VII.	———	Half Tangents.	S.T.
VIII.	———	Longitude.	Lon.
IX.	———	Latitude.	Lat.
X.	———	Hours.	Ho.
XI.	———	Inclinations.	In. Mer.

Of the Lines of equal Parts.

LINES of equal parts are of two sorts, *viz.* simply divided, and diagonally divided. Pl. V.

I. *Simply divided.* Draw 3 lines parallel to one another, at unequal distances, (Fig. 14.) and of any convenient length; divide this length into what number of equal parts is thought necessary, allowing some certain number of these parts to an inch, such as 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c. which divisions distinguish by

I lines

lines drawn acroſs the three parallels. Divide the left hand diviſion into 10 equal parts, which diſtinguiſh by lines drawn acroſs the lower parallels only; but, for diſtinction ſake, let the 5th diviſion be ſomewhat longer than the others: and it may not be inconvenient to divide the ſame left-hand diviſion into 12 equal parts, which are laid down on the upper parallel line, having the 3d, 6th, and 9th diviſions diſtinguiſhed by longer ſtrokes than the reſt, whereof that at the 6th diviſion make the longeſt.

There are, for the moſt part, ſeveral of theſe ſimply divided ſcales put on rulers one above the other, with numbers on the left hand, ſhewing in each ſcale, how many equal parts an inch is divided into; ſuch as 20, 25, 30, 35, 40, 45, *&c.* and are ſeverally uſed, as the plan to be expreſſed ſhould be larger or ſmaller.

The uſe of theſe lines of equal parts, is to lay down any line expreſſed by a number of two places or denominations, whether decimally, or duodecimally divided; as leagues, miles, chains, poles, yards, feet, inches, *&c.* and their tenth parts, or twelfth parts: thus, if each of the diviſions be reckoned 1, as 1 league, mile, chain, *&c.* then each of the ſubdiviſions will expreſs $\frac{1}{10}$ part thereof; and if each of the large diviſions be called 10, then each ſmall one will be 1; and if the large diviſions be 100, then each ſmall one will be 10, *&c.*

Therefore to lay off a line 8 $\frac{7}{10}$, 87, or 870 parts, let them be leagues, miles, chains, *&c.* ſet one point of the compaſſes on the 8th of the large diviſions, counting from the left hand towards the right, and open the compaſſes, till the other point falls on the 7th of the ſmall diviſions, counting from the right hand towards the left, then are the compaſſes opened to expreſs a line of 8 $\frac{7}{10}$, 87 or 870 leagues, miles, chains, *&c.* and bears ſuch proportion in the plan, as the line meaſured does to the thing repreſented.

But if a length of feet and inches was to be expressed, the same large divisions may represent the feet, but the inches must be taken from the upper part of the first division, which (as before noted) is divided into 12 equal parts.

Thus, if a line of 7 feet 5 inches was to be laid down; set one point of the compasses on the 5th division among the 12, counting from the right hand towards the left, and extend the other to 7, among the large divisions, and that distance laid down in the plan, shall express a line of 7 feet 5 inches: and the like is to be understood of any other dimensions.

II. *Diagonally divided.* Draw eleven lines parallel to each other, and at equal distances; divide the upper of these lines into such a number of equal parts, as the scale to be expressed is intended to contain, and from each of these divisions draw perpendiculars through the eleven parallels, (Fig. 15.) subdivide the first of these divisions into 10 equal parts, both in the upper and lower lines; then each of these subdivisions may be also subdivided into 10 equal parts, by drawing diagonal lines; *viz.* from the 10th below, to the 9th above; from the 9th below, to the 8th above; from the 8th below, to the 7th above, *&c.* till from the 1st below to the 0th above, so that by these means one of the primary divisions on the scale, will be divided into 100 equal parts.

There are generally two diagonal scales laid on the same plane or face of the ruler, one being commonly half the other. (Fig. 15.)

The use of the diagonal scale is much the same with the simple scale; all the difference is, that a plan may be laid down more accurately by it: because in this, a line may be taken of three denominations; whereas from the former, only two could be taken.

Now from this construction it is plain, if each of the primary divisions represent 1, each of the first subdivisions will express $\frac{1}{10}$ of 1; and each of the

second

ſecond ſubdiviſions, (which are taken on the diagonal lines, counting from the top downwards) will expreſs $\frac{1}{10}$ of the former ſubdiviſions, or a 100th of the primary diviſions; and if each of the primary diviſions expreſs 10, then each of the firſt ſubdiviſions will expreſs 1, and each of the 2d, $\frac{1}{10}$; and if each of the primary diviſions repreſent 100, then each of the firſt ſubdiviſions will be 10; and each of the 2d will be 1, &c.

THEREFORE to lay down a line, whoſe length is expreſs'd by 347, 34 $\frac{7}{10}$ or 3 $\frac{47}{100}$ whether leagues, miles, chains, &c.

ON the diagonal line, joined to the 4th of the firſt ſubdiviſions, count 7 downwards, reckoning the diſtance of each parallel 1; there ſet one point of the compaſſes, and extend the other, till it falls on the interſection of the third primary diviſion with the ſame parallel in which the other foot reſts, and the compaſſes will then be opened to expreſs a line of 347, 34 $\frac{7}{10}$; or 3 $\frac{47}{100}$, &c.

THOSE who have frequent occaſion to uſe ſcales, perhaps will find, that a ruler with the 20 following ſcales on it, *viz.* 10 on each face, will ſuit more purpoſes than any ſet of ſimply divided ſcales hitherto made public, on one ruler.

One Side	The diviſions	10, 11, 12, 13½, 15, 16½, 18, 20, 22, 25.
Other Side	to an inch	28, 32, 36, 40, 45, 50, 60, 70, 85, 100.

THE left hand primary diviſion, to be divided into 10 and 12 and 8 parts; for theſe ſubdiviſions are of great uſe in drawing the parts of a fortreſs, and of a piece of cannon.

IT will here be convenient to ſhew, how any plan expreſſed by right lines and angles, may be delineated by the ſcales of equal parts, and the protractor.

PROBLEM VIII.

Three adjacent things in any right lined triangle being given, to form the plan thereof.

Exam. *Suppose a triangular field*, ABC, (Fig. 16.) the side AB=327 yards; AC=208 yards; and the angle at A=$44\frac{1}{2}$ degrees.

Construction. Draw a line AB at pleasure; then from the diagonal scale take 327 between the points of the compasses, and lay it from A to B; set the center of the protractor to the point A, lay off $44\frac{1}{2}$ degrees, and by that mark draw AC: take with the compasses from the same scale 208, lay it from A to C, and join CB; so shall the parts of the triangle ABC, in the plan, bear the same proportion to each other, as the real parts in the field do.

The side CB may be measured on the same scale from which the sides AB, AC, were taken: and the angles at B and C may be measured by applying the protractor to them as shewn at problem IV.

If two angles and the side contained between them were given.

Draw a line to express the side; (as before) at the ends of that line, point off the angles, as observed in the field; lines drawn from the ends of the given line through those marks, shall form a triangle similar to that of the field.

PROBLEM IX.

Five adjacent things, sides and angles, in a right lin'd quadrilateral, being given, to lay down the plan thereof, Fig. 17.

Exam.

EXAM. *Given* ∠ * A = 70°; AB = 215 links; ∠ B = 115°; BC = 596 links; ∠ C = 114°.

CONSTRUCTION. Draw AD at pleasure; from A draw AB, so as to make with AD an angle of 70°: make AB = 215 (taken from the scales); from B, draw BC, to make with AB an angle of 115°: make BC = 596; from C, draw CD, to make with CB an angle of 114°, and by the intersection of CD with AD, a quadrilateral will be form'd similar to the figure in which such measures could be taken as are expressed in the example.

IF 3 of the things were sides, the plan might be formed with equal ease.

FOLLOWING the same method, a figure of many more sides may be delineated; and in this manner, or some other like to it, do some surveyors make their plans of surveys.

The Construction of the remaining Lines of the PLAIN SCALE.

PREPARATION. Fig. 18. Pl. VI.

DESCRIBE a circumference with any convenient radius, and draw the diameters AB, DE, at right angles to each other; continue BA at pleasure towards F; through D, draw DG parallel to BF; and draw the chords BD, BE, AD, AE. Circumscribe the circle with the square HMN, whose sides HM, MN, shall be parallel to AB, ED.

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* This mark or character ∠, signifies *the angle.*
This mark = signifies *equal to.*
By links, is meant the $\frac{1}{100}$th part of a chain of four poles or of 66 yards long.

I. *To construct the Line of Chords* *.

DIVIDE the arc AD into 90 equal parts; mark the 10th divisions with the figures 10, 20, 30, 40, 50, 60, 70, 80, 90; on D, as a center, with the compasses, transfer the several divisions of the quadrantal arc, to the chord AD, which marked with the figures corresponding, will become a line of chords.

Note, In the construction of this, and the following scales, only the primary divisions are drawn; the intermediate ones are omitted, that the figure may not appear too much crouded.

* The chord of an arc, is a right line drawn from one end of the arc to the other end.

II. *The Line of Rhumbs* †.

DIVIDE the arc BE into 8 equal parts, which mark with the figures 1, 2, 3, 4, 5, 6, 7, 8; and divide each of those parts into quarters; on B, as a center, transfer the divisions of the arc to the chord BE, which marked with the corresponding figures, will be a line of rhumbs.

† The rhumbs here, are the chords answering to the points of the mariners compass, which are 32 in the whole circle, or 8 in the quarter circle.

III. *The Line of Sines* ‡.

THROUGH each of the divisions of the arc AD, draw right lines parallel to the radius AC; and CD will be divided into a line of sines which are to be numbered

‡ The *sine of an arc*, is a right line drawn from one end of an arc perpendicular to a radius drawn to the other end.

And the *versed sine*, is the part of the radius lying between the arc and its right sine.

from C to D for the right sines; and from D to C for the versed sines. The versed sines may be continued to 180 degrees by laying the divisions of the radius CD, from C to E.

IV. *The Line of Tangents* *.

A RULER on C, and the several divisions of the arc AD, will intersect the line DG, which will become a line of tangents, and is to be figured from D to G with 10, 20, 30, 40, *&c.*

* The *tangent of an arc*, is a right line touching that arc at one end, and terminated by a secant drawn through the other end.

V. *The Line of Secants* †.

THE distances from the center C to the divisions on the line of tangents being transferred to the line AF from the centre C, will give the divisions of the line of secants; which must be numbered from A towards F, with 10, 20, 30, *&c.*

† The *secant of an arc*, is a right line drawn from the centre through one end of an arc, and limited by the tangent of that arc.

VI. *The Line of Half-Tangents (or the Tangents of half the Arcs).*

A RULER on E, and the several divisions of the arc AD, will intersect the radius CA, in the divisions of the semi, or half tangents; mark these with the corresponding figures of the arc AD.

THE semi-tangents on the plane scales are generally continued as far as the length of the ruler they are laid on will admit; the divisions beyond 90° are found by dividing the arc AE like the arc AD, then laying a ruler by E and these divisions of the arc AE, the divisions

sions of the semi-tangents above 90 degrees will be obtained on the line CA continued.

VII. *The Line of Longitude.*

DIVIDE AH, into 60 equal parts; through each of these divisions, parallels to the radius AC, will intersect the arc AE, in as many points; from E as a centre, the divisions of the arc EA, being transferred to the chord EA, will give the divisions of the line of longitude.

VIII. *The Line of Latitude.*

A RULER on A, and the several divisions of the sines on CD, will intersect the arc BD, in as many points; on B as a centre, transfer the intersections of the arc BD, to the right line BD; number the divisions from B to D, with 10, 20, 30, &c. to 90; and BD will be a line of latitude.

IX. *The Line of Hours.*

BISECT the quadrantal arcs BD, BE, in *a*, *b*; divide the quadrantal arc *ab* into 6 equal parts, (which gives 15 degrees for each hour) and each of these into 4 others; (which will give the quarters.) A ruler on C, and the several divisions of the arc *ab*, will intersect the line MN in the hour, &c. points, which are to be marked as in the figure.

X. *The Line of Inclinations of Meridians.*

BISECT the arc EA into *c*; divide the quadrantal arc *bc* into 90 equal parts; lay a ruler on C and the several divisions of the arc *bc*, and the intersections of the line HM will be the divisions of a line of inclinations of meridians.

SECT.

Sect. VIII.

The uses of some of the Lines on the Plain Scale.

I. *Of the Line of Chords.* Pl. VI.

One of the uses of the line of chords is *to lay down a proposed angle, or to measure an angle already laid down.* Thus, to draw a line AC, that shall make with the line AB an angle containing a given number of degrees. (suppose 36.) Figure 19.

On A, as a centre, with a radius equal to the chord of 60 degrees, describe the arc BC; on this arc, lay the chord of the given number of degrees from the intersection B, to C; draw AC, and the angle BAC will contain the given number of degrees.

Note, Degrees taken from the chords are always to be counted from the beginning of the scale.

The degrees contained in an angle already laid down, may be measured thus. Fig. 19.

On A as a centre, describe an arc BC with the chord of 60 degrees; the distance BC, measured on the chords, will give the number of degrees contained in the angle BAC.

If the number of degrees are more than 90; they must be taken from, or measured by the chords, at twice; thus if 140 degrees were to be protracted, 70° may be taken from the chords, and those degrees laid off twice upon the arc described with a chord of 60 degrees.

Note, Degrees are generally denoted by a small ° put over them.

II. *Of the Line of Rhumbs.*

Their use is to delineate or measure a ship's course; which is the angle made by a ship's way and the meridian. Now

Now having the points and $\frac{1}{4}$ points of the compaſs contained in any courſe; draw a line AB (fig. 19.) for the meridian; on A as a centre, with a chord of 60° deſcribe an arc BC; take the number of points and $\frac{1}{4}$ points from the ſcale of rhumbs, counting from 0, and lay this diſtance on the arc BC, from the interſection B to C; draw AC, and the angle BAC ſhall repreſent the ſhip's courſe.

III. *The uſe of the Line of Longitude.*

If any two meridians be diſtant one degree or 60 geographical miles, under the equator, their diſtance will be leſs than 60 miles in any latitude between the equator and the pole.

Now let the line of longitude be put on the ſcale cloſe to the line of chords, but inverted; that is, let 60° in the ſcale of longitude be againſt 0° in the chords, and 0° degrees longitude againſt 90° chords. Then mark any degree of latitude counted on the chords; and oppoſite thereto, on the line of longitude, will be the miles contain'd in one degree of longitude, in that latitude.

Thus 57,95 miles, make 1 degree of longitude in the latitude of 15 degrees; 45,97 miles, in latitude 40 degrees; 36,94 miles, in latitude 52 degrees; 30 miles, in latitude 60 degrees, &c.

But as the fractional parts are not very obvious on ſcales, here follows a table ſhewing the miles in one degree of longitude to every degree of latitude.

This table is computed, upon the ſuppoſition of the earth being ſpherical, by the following proportion.

As the radius is to the coſine of any latitude, ſo is the miles of longitude under the equator to the miles of longitude in that latitude.

Every perſon who is deſirous of acquiring mathematical knowledge, ſhould have a table of the logarithms of numbers, ſines, tangents, and ſecants; moſt of

of the treatises of navigation, and some other books, have these tables; but the most useful and esteemed are *Sherwin*'s mathematical tables.

A TABLE, *shewing the Miles in one Degree of Longitude to every Degree of Latitude.*

D. L.	Miles.	D. L.	Miles.	D. L.	Miles.
1	59,99	31	51,43	61	29,09
2	59,96	32	50,88	62	28,17
3	59,92	33	50,32	63	27,24
4	59,85	34	49,74	64	26,30
5	59,77	35	49,15	65	25,36
6	59,67	36	48,54	66	24,41
7	59,56	37	47,92	67	23,44
8	59,42	38	47,28	68	22,48
9	59,26	39	46,63	69	21,50
10	59,09	40	45,97	70	20,52
11	58,89	41	45,28	71	19,53
12	58,69	42	44,59	72	18,54
13	58,46	43	43,88	73	17,54
14	58,22	44	43,16	74	16,54
15	57,95	45	42,43	75	15,53
16	57,67	46	41,68	76	14,52
17	57,38	47	40,92	77	13,50
18	57,06	48	40,15	78	12,48
19	56,73	49	39,36	79	11,45
20	56,38	50	38,57	80	10,42
21	56,02	51	37,76	81	9,38
22	55,63	52	36,94	82	8,35
23	55,23	53	36,11	83	7,32
24	54,81	54	35,27	84	6,28
25	54,38	55	34,41	85	5,23
26	53,93	56	33,55	86	4,18
27	53,46	57	32,68	87	3,14
28	52,96	58	31,79	88	2,09
29	52,47	59	30,90	89	1,05
30	51,96	60	30,00	90	0,00

The uses of the scales of sines, tangents, secants, and half tangents, are to find the poles and centers of the several circles represented in the orthographical and stereographical projection of the sphere; which are reserved until the explanation and use of the lines of the same name on the sector are shewn.

The lines of latitudes, hours, and inclinations of meridians, are applicable to the practice of dialing; on which there are several treatises extant, which may be consulted.

Sect. IX.

Of the Sector.

A Sector is a figure form'd by two radius's of a circle, and that part of the circumference comprehended between the two radius's.

The instrument called a sector, consists of two rulers moveable round an axis or joint, from whence several scales are drawn on the faces of the rulers.

The two rulers are called legs, and represent the radii, and the middle of the joint expresses the center.

The scales generally put on sectors, may be distinguished into single, and double.

The single scales are such as are commonly put on plain scales, and from whence dimensions or distances are taken as have been already directed.

The double scales are those which proceed from the center; each scale is laid twice on the same face of the instrument, *viz.* once on each leg: From these scales, dimensions or distances are to be taken, when the legs of the instrument are in an angular position, as will be shewn hereafter.

The

The Scales commonly put on the best Sectors, are

		a line of		mark'd
Single	1	Inches, each Inch divided into 8 and 10 parts.		
	2	Decimals, containing an 100 parts.		
	3	Chords,		Cho.
	4	Sines,		Sin.
	5	Tangents,		Tang.
	6	Rhumbs,		Rum.
	7	Latitude,		Lat.
	8	Hours,		Hou.
	9	Longitude,		Lon.
	10	Inclin. Merid.		In.Me
	11	the Logarithms of	Numbers,	Num.
	12		Sines,	Sin.
	13		Versed Sines,	V.Sin.
	14		Tangents,	Tan.

		a line of	mark'd
Double	1	Lines, or of equal parts,	Lin.
	2	Chords.	Cho.
	3	Sines.	Sin.
	4	Tangents to 45°	Tan.
	5	Secants,	Sec.
	6	Tangents to above 45°	Tan.
	7	Polygons,	Pol.

THE manner in which these scales are disposed of on the sector, is best seen in the plate fronting the title page.

THE scales of lines, chords, sines, tangents, rhumbs, latitudes, hours, longitude, incl. merid. may be used, whether the instrument is shut or open, each of these scales being contained on one of the legs only. The scales of inches, decimals, log. numbers, log. sines, log. versed sines and log. tangents, are to be used with the sector quite opened, part of each scale lying on both legs.

THE double scales of lines, chords, sines, and lower tangents, or tangents under 45 degrees, are all of the same radius or length; they begin at the center of the instrument, and are terminated near the other extremity of each leg; *viz.* the lines at the division

division 10, the chords at 60, the sines at 90, and the tangents at 45; the remainder of the tangents, or those above 45°, are on other scales beginning at $\frac{1}{4}$ of the length of the former, counted from the center, where they are marked with 45, and run to about 76 degrees.

THE secants also begin at the same distance from the center, where they are marked with 10, and are from thence continued to as many degrees as the length of the sector will allow, which is about 75°.

THE angles made by the double scales of lines, of chords, of sines, and of tangents to 45 degrees, are always equal.

AND the angles made by the scales of upper tangents, and of secants, are also equal; and sometimes these angles are made equal to those made by the other double scales.

THE scales of polygons are put near the inner edge of the legs, their beginning is not so far removed from the center, as the 60 on the chords is: Where these scales begin, they are mark'd with 4, and from thence are figured backwards, or towards the center, to 12.

FROM this disposition of the double scales, it is plain, that those angles which were equal to each other, while the legs of the sector were close, will still continue to be equal, although the sector be opened to any distance it will admit of.

SECT.

SECT. X.

Of the Construction of the Single Scales.

I. *The Scale of Inches.*

THIS ſcale, which is laid cloſe to the edge of the ſector, and ſometimes on the edge, contains as many inches as the inſtrument will receive when opened: Each inch is uſually divided into 8 equal parts, and alſo into 10 equal parts.

II. *The Decimal Scale.*

THIS ſcale lies next to the ſcale of inches; it is of the ſame length of the ſector when opened, and is divided into 10 equal parts, or primary diviſions; and each of theſe into 10 other equal parts; ſo that the whole is divided into 100 equal parts. And where the ſector is long enough, each of the ſubdiviſions is divided into two, four, or five parts; and by this decimal ſcale, all the other ſcales, that are taken from tables, may be laid down.

THE length of a ſector is uſually underſtood when it is ſhut, or the legs cloſed together. Thus a ſector of ſix inches when ſhut, makes a ruler of twelve inches when opened, and a foot ſector, is two feet long when quite opened.

III. *The Scales of Chords, Rhumbs, Sines, Tangents. Hours, Latitudes, Longitudes, and Inclination of Meridians;*

ARE ſuch as have been already deſcribed in the account of the plane ſcale.

IV. *The Scale of Logarithmic Numbers.*

THIS ſcale, commonly called the artificial numbers, and by ſome the *Gunter*'s ſcale, or *Gunter*'s * line, is a ſcale expreſſing the logarithms of common numbers, taken in their natural order. To lay down the diviſions in the beſt manner, there is neceſſary a good table of logarithms, (ſuppoſe *Sherwin*'s,) and a ſcale of equal parts, accurately divided, and of ſuch a length, that 20 of the primary diviſions ſhall make the whole length of the intended ſcale of numbers, or logarithm ſcale.

The Conſtruction.

1. FROM the ſcale of equal parts, take the firſt 10 of the primary diviſions, and lay this diſtance down twice on the log. ſcale, making two equal intervals; marking the firſt point 1, the ſecond 1, (or rather 10) and the third 10, (or rather 100.)

2. FROM the ſcale of equal parts, take the diſtances expreſſed by the logs. of the numbers, 2, 3, 4, 5, 6, 7, 8, 9, reſpectively, (rejecting the indices:) lay theſe diſtances on each interval of the log. ſcale, between the marks 1 & 10, 10 & 100, reckoning each diſtance from the beginning of its interval, *viz.* from 1, and from 10, and mark theſe diſtances with the figures 2, 3, 4, 5, 6, 7, 8, 9, in order.

THUS the firſt three figures of the logarithms of 2, 3, 4, 5, 6, 7, 8, 9, are, 301, 477, 602, 699, 778, 845, 903, 954; theſe are the numbers that are to be taken from the ſcale of equal parts, and laid

* From Mr. *Edmund Gunter*, the Inventor: Aſtronomy-Profeſſor in *Greſham College*, Anno 1624.

down

down in each interval, obſerving that the extent for each is to be applied from the beginning of the intervals.

3. The diſtances expreſſing the logs. of the numbers between 10 & 20, 20 & 30, 30 & 40, 40 & 50, 50 & 60, 60 & 70, 70 & 80, 80 & 90, 90 & 100, (rejecting the indices) are alſo to be taken from the ſcale of equal parts, and laid on the log. ſcale, in each of the primary intervals, between the marks 1 & 2, 2 & 3, 3 & 4, 4 & 5, 5 & 6, 6 & 7, 7 & 8, 8 & 9, 9 & 10, reſpectively; reckoning each diſtance from the beginning of its reſpective primary interval.

4. The laſt ſubdiviſions of the ſecond primary interval are to be divided into others, as many as the ſcale will admit of, which is done by laying down the logarithms of ſuch intermediate diviſions, as it ſhall be thought proper to introduce.

V. *The Scale of Logarithm Sines.*

1. From the ſcale of equal parts, take the diſtances expreſſed by the arithmetical complements * of the logarithmic ſines, (or the ſecants of the complements) of 80, 70, 60, 50, 40, 30, 20, 10, degrees reſpectively; rejecting the indices; and theſe diſtances, lay on the ſcale of log. ſines, reckoning each from the mark intended to expreſs 90 degrees.

Thus. To the ſines of 80°, 70°, 60°, 50°, 40°, 30°, 20°, 10°, the three firſt figures of the arithmetical complements of their logarithms, are, 007, 026, 063, 115, 192, 301, 466, 760; theſe are the numbers to be taken from the ſcale of equal parts, uſed for

* By the arithmetical complement of any ſine, tangent, *&c.* is meant the remainder, when that ſine, tangent, *&c.* is ſubſtracted from radius, or 10,000000, *&c.*

laying down the logarithms of numbers, and every extent of the compasses is to be laid from the right hand towards the left, beginning at the point chose for 90°, which usually stands directly under the end of the line of numbers.

2. In the same manner, lay off the degrees under 10; and also, the degrees intermediate to those of 10, 20, 30, *&c.*

3. Lay down as many of the multiples of 5 minutes, as may conveniently fall within the limits of those degrees which will admit of such subdivisions of minutes.

VI. *The Scale of Logarithmic Tangents.*

1. This scale, as far as 45 degrees, is constructed, in every particular, like that of the log. sines; using the arithmetical complements of the log. tangents.

2. The degrees above 45, are to be counted backwards on the scale: Thus 40 on the scale, represents both 40 degrees, and 50 degrees; 30 on the scale, represents both 30 degrees, and 60 degrees; and the like of the other mark'd degrees, and also of their intermediate ones.

VII. *The Logarithmic versed Sines.*

1. From the scale of equal parts, take the arithmetical complements of the logarithm co-sines, (or the secants of the complements) of 5, 10, 15, 20, 25, 30, 35, 40, *&c.* degrees; (rejecting the indices,) and the double of these distances, respectively, laid on the scale (intended) for the log. versed sines, will give the divisions expressing 10, 20, 30, 40, 50, 60, 70, 80, *&c.* degrees; to as many as the length of the scale will take in.

2. Between every distance of 10 degrees, introduce as many degrees, ½ degrees; ⅓ degrees; ¼ degrees, *&c.* as the intervals will admit.

The

The scales of the logarithms of numbers, sines, versed sines, and tangents, should have one common termination to one end of each scale; that is, the 10 on the numbers, the 90 on the sines, the 0 on the versed sines, and the 45 on the tangents, should be opposite to each other: The other end of each of the scales of sines, versed sines, and tangents, will run out beyond the beginning (mark'd 1) of the numbers; nearly opposite to which, will be the divisions representing 35 minutes on the sines and tangents, and $168\frac{1}{2}$ degrees, on the versed sines.

Sect. XI.

Of the Construction of the Double Scales.

I. *Of the Line of Lines.*

THIS is only a scale of equal parts, whose length is adapted to that of the legs of the sector: Thus in the six inch sector, the length is about $5\frac{3}{4}$ inches.

The length of this scale is divided into 10 primary divisions; each of these into 10 equal secondary parts; and each secondary division, into 4 equal parts.

Hence on any sector it will be easy to try if this line is accurately divided: Thus. Take between the compasses any number of equal parts from this line, and apply that distance to all the parts of the line; and if the same number of divisions are contained between the points of the compasses in every application, the scale may be received as perfect.

II. *Of the Line of Sines.*

1. MAKE the whole length of this ſcale, equal to that of the line of lines.

2. FROM the ſcale of the line of lines, take off ſeverally, the parts expreſſed by the numbers in the tables (ſuppoſe *Sherwin*'s) of the natural ſines, correſponding to the degrees, or to the degrees and minutes, intended to be laid on the ſcale.

3. LAY down theſe diſtances ſeverally on the ſcale, beginning from the center; and this will expreſs a ſcale of natural ſines.

EXAM. *To lay down* 35° 15′; *whoſe natural ſine found in the tables is* 57714, *&c.*

TAKE this number as accurately as may be, from the line of lines, counting from the center; and this diſtance will reach from the beginning of the ſines, at the center of the inſtrument, to the diviſion expreſſing 35° 15′; and ſo of the reſt.

IN ſcales of this length, it is cuſtomary to lay down diviſions, expreſſing every 15 minutes, from 0 degrees to 60 degrees; between 60 and 80 degrees, every half degree is expreſſed; then every degree to 85; and the next, is 90 degrees.

III. *Of the Scale of Tangents.*

THE length of this ſcale is equal to that of the line of lines, and the ſeveral diviſions thereon (to 45 degrees) are laid down from the tables and line of lines, in the ſame manner as has been deſcribed in the ſines; obſerving to uſe the natural tangents in the tables.

IV. *Of*

IV. *Of the Scale of upper Tangents.*

This ſcale is to be laid down, by taking $\frac{1}{4}$ of ſuch of the natural tabular tangents above 45 degrees, as are intended to be put on the ſcale.

Although the poſition of this ſcale on the ſector reſpects the center of the inſtrument, yet its beginning, at 45 degrees, is diſtant from the center, $\frac{1}{4}$ of the length or radius of the lower tangents.

V. *Of the Scale of Secants.*

The diſtance of the beginning of this ſcale, from the center, and the manner of laying it down, is juſt the ſame as that of the upper tangents; only in this, the tabular ſecants are to be uſed.

VI. *Of the Scale of Chords.*

1. Make the length of this ſcale, equal to that of the ſines; and let the diviſions to be laid down, expreſs every 15 minutes from 0 degrees to 60 degrees.

2. Take the length of the ſine of half the degrees and minutes, for every diviſion to be laid down, (as before directed in the ſcale of ſines;) and twice this length, counted from the center, will give the diviſions required.

Thus, twice the length of the ſine 18° 15′, will give the chord of 36° 30′; and in the ſame manner for the reſt.

VII. *Of the Scale of Polygons.*

This ſcale uſually takes in the ſides of the polygons from 6 to 12 ſides incluſive: The diviſions are laid down, by taking the lengths of the chords of

the angles at the center of each polygon; and these distances are laid from the center of the instrument.

But it is best to have the polygons of 4 and 5 sides also introduced; and then this line is constructed from a scale of chords, where the length of 90 degrees is equal to that of 60 degrees of the double scale of chords on the sector.

In the place of some of the double scales here described, there are found other scales on the old sectors, and also on some of the modern *French* ones, such as, scales of superficies, of solids, of inscribed bodies, of metals, *&c.* But these seem to be justly left out on the sectors, as now constructed, to make room for others of more general use: However, these scales, and some others, of use in gunnery, shall hereafter be described in a tract on the use of the gunners callipers.

Sect. XII.

Of the Uses of the Double Scale.

In the following account of the uses, as there will frequently occur the terms *lateral distance*, and *transverse distance*; it will be proper to explain what is meant by those terms.

Lateral distance, is a distance taken by the compasses on one of the scales only, beginning at the center of the sector.

Transverse distance, is the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position: That is, one foot of the compasses is set on a division in a scale on one leg of the sector, and the other foot is extended to the like division in the

the ſcale of the ſame name on the other leg of the ſector.

It muſt be obſerved, that each of the ſectoral ſcales have three parallel lines, acroſs which the diviſions of the ſcale are marked: Now in taking tranſverſe diſtances, the points of the compaſſes muſt be always ſet on the inſide line, or that line next the inner edge of the leg; for this line only in each ſcale runs to the center.

Some Uſes of the Line of Lines.

PROBLEM X.

To two given lines AB = 2, BC = 6; *to find a third proportional.* Plate VI. Fig. 20.

Operation. 1. Take between the compaſſes, the lateral diſtance of the ſecond term, (*viz.* 6.)

2. Set one point on the diviſion expreſſing the firſt term (*viz.* 2.) on one leg, and open the legs of the ſector till the other point will fall on the correſponding diviſion on the other leg.

3. Keep the legs of the ſector in this poſition; take the tranſverſe diſtance of the ſecond term, (*viz.* 6.) and this diſtance is the third term required.

4. This diſtance meaſured laterally, beginning from the center, will give (18) the number expreſſing the meaſure of the third term: For 2 : 6 :: 6 : 18.

Or, Take the diſtance 2 laterally, and apply it tranſverſely to 6 and 6 (the ſector being properly opened), then the tranſverſe diſtance at 2 and 2 being taken with the compaſſes and applied laterally from the center of the ſector on the ſcale of lines, will give ,66$\frac{2}{3}$ = $\frac{2}{3}$, the third term when the proportion is decreaſing: For 6 : 2 :: 2 : $\frac{2}{3}$.

Note, If the legs of the ſector will not open ſo far as to let the lateral diſtance of the ſecond term fall between the diviſions expreſſing the firſt term; then take

take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any aliquot part of the ſecond term, (ſuch as will conveniently fall within the opening of the ſector) and make ſuch part, the tranſverſe diſtance of the firſt term; then if the tranſverſe diſtance of the ſecond term be multiplied by the denominator of the part taken of the ſecond term, the product will give the third term.

PROBLEM XI.

To three given lines AB = 3, BC = 7, CD = 10; *to find a fourth proportional.* Plate VI. Fig. 21.

OPERATION. Open the legs of the ſector, until the tranſverſe diſtance of the firſt term, (3) be equal to the lateral diſtance of the ſecond term, (7) or to ſome part thereof; then will the tranſverſe diſtance of the third term, (10) give the fourth term, ($23\frac{1}{3}$) required; or, ſuch a ſubmultiple thereof as was taken of the ſecond term: For 3 : 7 :: 10 : $23\frac{1}{3}$

OR, Set the lateral diſtance 7 tranſverſely from 10 to 10 (opening the ſector properly); then the tranſverſe diſtance at 3 and 3 taken and applied laterally, will give $2\frac{1}{10}$: For 10 : 7 :: 3 : $2\frac{1}{10}$.

FROM this problem is readily deduced, how to increaſe or diminiſh a given line, in any aſſigned proportion.

EXAM. *To diminiſh a line of* 4 *inches, in the proportion of* 8 *to* 7.

1. OPEN the ſector until the tranſverſe diſtance of 8 and 8, be equal to the lateral diſtance of 7.

2. MARK the point to where 4 inches will reach, as a lateral diſtance taken from the center.

3. THE tranſverſe diſtance, taken at that point, will be the line required.

IF the given line, ſuppoſe 12 inches, ſhould be too long for the legs of the ſector, take $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. part of the given line for the lateral diſtance;

 and

and the corresponding transverse distance, taken twice, or thrice, or four times, *&c.* will be the line required.

PROBLEM XII.

To open the sector so, that the two scales of lines shall make a right angle.

Operation. Take the lateral distance from the center to the division marked 5 between the points of the compasses, and set one foot on the division marked 4 on one of the scales of lines, and open the legs of the sector till the other foot falls on the division marked 3 on the other scale of lines, and then will those scales stand at right angles to one another.

For the lines 3, 4, 5, or any of their multiples, constitute a right angle triangle.

PROBLEM XIII.

To two right lines given, to find a mean proportional. Suppose the lines 40 *and* 90.

Operation. 1st. Set the two scales of lines at right angles to one another.

2d. Find the half sum of the given lines $(= \frac{90+40}{2} = 65)$; also find the half difference of those lines $(= \frac{90-40}{2} = 25)$.

3d. Take, with the compasses, the lateral distance of the half sum (65), and apply one foot to the half difference (25), the other foot transversely will reach to (60) the mean proportional required: For 40 : 60 :: 60 : 90.

PROBLEM

PROBLEM XIV.

To divide a given line into any proposed number of equal parts: (suppose 9).

Make the length of the given line, or some known part thereof, a transverse distance to 9 and 9: Then will the transverse distance of 1 and 1, be the $\frac{1}{9}$ part thereof; or such a submultiple of the $\frac{1}{9}$ part, as was taken of the given line.

Or the $\frac{1}{9}$ part, will be the difference between the given line, and the transverse distance of 8 and 8.

The latter of these methods is to be preferred when the part required falls near the center of the instrument.

To this problem may be referred the method of making a scale of a given length, to contain a given number of equal parts.

The practice of this is very useful to those who have occasion to take copies of surveys of lands; draughts of buildings, whether civil or military; and in every other case, where drawings are to be made to bear a given proportion to the things they represent.

Exam. *Suppose the scale to the map of a survey is 6 inches long, and contains 140 poles; required to open the sector so, that a corresponding scale may be taken from the line of lines.*

Solution. Make the transverse distance 7 and 7 (or 70 and 70, *viz.* $\frac{140}{2}$) equal to three inches ($= \frac{6}{2}$); and this position of the line of lines will produce the given scale.

If it was required to make a scale of 140 poles, and to be only two inches long.

Solution. Make the transverse distance of 7 and 7 equal to one inch, and the scale is made.

Exam.

Exam. II. *To make a ſcale of 7 inches long contain 180 fathoms.*

Solution. Make the tranſverſe diſtance of 9 and 9 equal to $3\frac{1}{2}$ inches, and the ſcale is made.

Exam. III. *To make a ſcale which ſhall expreſs 286 yards, and be 18 inches long.*

Solution. Make the $\frac{1}{3}$ of 18 inches (or 6 inches) a tranſverſe diſtance to the $\frac{1}{3}$ of 286 ($= 95\frac{1}{3}$) and the ſcale is made.

Or, Make the $\frac{1}{4}$ of 18 inches ($= 4\frac{1}{2}$ inches) a tranſverſe diſtance to $\frac{1}{4}$ of 286 ($= 71\frac{1}{2}$), and the ſcale is made.

Exam. IV. *To divide a given line (ſuppoſe of 5 inches) into any aſſigned proportion (as of 4 to 5).*

Solution. Take (5 inches) the length of the given line, between the compaſſes, and make this a tranſverſe diſtance to (9 and 9) the ſum of the propoſed parts; then the tranſverſe diſtances of the aſſigned numbers (4 and 5) will be the parts required.

PROBLEM XV.

The uſe of the line of lines in drawing the orders of Civil Architecture.

In this place it is intended to give ſo much of Architecture as may enable a beginner to draw any one of the orders; but that the following precepts may be rightly underſtood, it will be proper to explain a few of the terms.

Definitions.

1. Architecture is the art of building well; and has for its object the Convenience, Strength, and Beauty of the building.

2. ORDER in Architecture, is generally underſtood as Ornament, and conſiſts of three grand parts, namely;

3. THE ENTABLATURE, which repreſents, or is, the weight to be ſupported.

4. THE COLUMN, that which ſupports any weight.

5. THE PEDESTAL or foot whereon the Column is ſet for its better ſecurity.

EACH of theſe parts conſiſts alſo of three parts.

6. THE *Pedeſtal* is composed of a BASE, or lower part, a DIE, and a CORNICE, or upper part.

7. THE *Column* is made up of a BASE, a SHAFT, which is a middle part, and a CAPITAL, the upper part.

8. THE *Entablature* conſiſts of an ARCHITRAVE, or lower part, a FREEZE, the middle part, and a CORNICE, the upper part.

So that an Order may be ſaid to conſiſt of nine large parts, each of which is made up of ſmaller parts called Members; whereof ſome are *Plane*, ſome *Curved*, either convex or concave, or convexo-concave.

PLANE members of different magnitude have different names.

9. A FILLET or *liſt* is the leaſt plane or flat member.

10. A PLINTH is that flat member at the bottom of the Pedeſtal, or of the baſe of the Column.

11. A PLATEBAND, that at the top of the Pedeſtal, or the upper member of the Architrave in the Entablature.

12. AN ABACUS, that at the top of the capital.

13. THE FACIÆ or faces are flat members in the Architrave.

14. THE CORONA is a large flat member in the Cornice.

THE Convex members are,

15. AN ASTRAGAL of a ſmall ſemicircular convexity.

16. THE

16. A Fusarole when an Astragal is cut into parts like beads.

17. A Torus a large semicircular convexity.

18. An Ovola nearly of a quadrantal convexity.

The Concave members are,

19. A Cavetto nearly of a quadrantal concavity.

20. A Scotie of a concavity nearly semicircular.

The Convexo-Concave members are a Cymaise and a Cima.

21. A Cymaise or *Ove*, that whose convex part projects most; and by workmen is usually called an Ogee.

22. A Cima that whose concave part projects most.

23. Soffit is the under part of the Crown of an Arch, or of the Corona of an Entablature.

24. Trigliphs *(i. e.* three channels) is an Ornament in the Freeze of the Doric Order.

25. Metops *(i. e.* between three's) is the space of the Freeze between two Trigliphs.

26. Modilions, or Mutules, are the brackets or ends of beams supporting the Corona. In the Corinthian Order they are generally carved into a kind of Scrol.

27. Dentels are an Ornament looking somewhat like a row of teeth; and are placed in the Cornice of the Entablature.

It is customary among Architects to estimate the heights and projections of all the parts of every order by the diameter of the column at the bottom of the shaft, which they call a module; and suppose it to consist of 60 equal parts, which are called minutes.

Of the Tuscan Order.

This order, which some writers liken to a strong robust labouring man, is the most simple and unadorned of any of the orders: The places most recommended to use it in, are country farm-houses, stables,

ſtables, gateways to inns, and places where plainneſs and ſtrength are reckoned moſt neceſſary: Though there are inſtances where this order has been applied to buildings of a more public and elegant nature.

The general proportions aſſigned by *Palladio*.

1. Height of the column equal to ſeven diameters, or modules.

2. Height of the entablature equal to one fourth of the column, wanting half a minute.

3. Height of the pedeſtal equal to one module.

4. The capital and baſe, each half a module.

5. Breadth of the baſe on a level is $1\frac{1}{6}$ module.

6. Breadth of the capital equal to one module.

7. Diminishing of the column is $\frac{1}{4}$ module.

8. Projection of the beams ſupporting the eaves is $1\frac{3}{4}$ modules.

9. In colonades, the diſtance of the columns in the clear is 4 modules.

10. In arches, and the columns ſet on pedeſtals,
   The diſtance of the columns from middle to middle is $6\frac{5}{12}$ modules.
   Height of the arch is $7\frac{2}{3}$ modules.
   Breadth of the pilaſter between the column and paſſage is 26 minutes.

The ovolo under the corona, in the cornice of the entablature, is commonly continued within the corona, giving it a reverſe bending in the ſoffit, ſomething like a cyma.

Of the Doric Order.

This order, ſuppoſed to be invented by *Dorus* a king of *Achaia*, may be likened to a well limbed genteel man; and although of a bold aſpect, yet not ſo ſturdy and ruſticly clad as the Tuſcan. Architects place this order indifferently in towns: But when they would decorate a country ſeat with it, the open champaign ſituation ſeems beſt for the reception of the Doric

Doric order; notwithstanding which, there are many fine buildings of this order in other situations, where they have a very pleasing effect.

The following general proportions are given by *Palladio*,

1. Height of the column from $7\frac{1}{2}$ to 8, and $8\frac{2}{3}$ modules.
2. Height of the entablature is one fourth of the column.
3. Height of the pedestal equal to $2\frac{1}{3}$ modules.
4. The Attic base is used with this order, it is half a module in height, and so is the capital.
5. Breadth of the column's base is $1\frac{1}{6}$ module.
6. Breadth of the capital is 1 module $17\frac{1}{2}$ minutes.
7. Diminishing of the column is 8 minutes.
8. In colonades, the distance of the columns in the clear is $2\frac{3}{4}$ modules.
9. In arches, and the column set on pedestals,
   Distance of the columns from middle to middle is $7\frac{1}{2}$ modules.
   Height of the arch to its soffit is $10\frac{1}{4}$ modules.
   Breadth of the pilasters is 26 minutes.

In the Doric order the architrave has two faces and a plinth; the upper face is ornamented with rows of six *drips* or *bells*, covered with a plain cap: The freeze is divided into trigliphs and metops: The breadths of the drips, cap and trigliphs are each $\frac{1}{7}$ module: The trigliphs consist of two channels, two half channels, and three voids; the breadths of the channels and voids are each 5 minutes: The axis of the column continued, runs through the middle void, leaving the drips three on each side: The metops, or distances between the trigliphs, are equal to the height of the freeze, and are commonly ornamented with trophies, arms, roses, &c.

HERE follows a table for the particular conftruction of the ornaments with which the architrave and freeze are enriched.

	Altitude.	Projection	Profile.
	Min.	Min.	Min.
Capital	5	16	3
Freeze	45	—	—
Trigliphs	40	15	$\frac{1}{2} + 2\frac{1}{2}$
Plinth	$4\frac{1}{2}$	16	3
Cap	$1\frac{2}{3}$	15	2
Drips	$3\frac{1}{3}$	15	2

THE column figned altitude gives the heights of the particular parts.

THAT figned projection fhews the breadths of thofe parts on each fide of the middle line of the column continued.

AND under the word profile ftand the numbers fhewing how far the feveral parts project beyond the planes or faces of the members on which they are made.

THE foffit of the corona in the cornice of the entablature, is ufually ornamented with drips correfponding to the trigliphs, and rofes, arms, *&c.* over the metops.

THE fhaft of the column is fometimes fluted; that is, cut into channels from top to bottom, the channels meeting one another in an edge, and are in number twenty.

Of the IONIC ORDER.

THIS order, which is taller and flenderer than the Doric, does not appear with fuch a mafculine ftrength; and

and is by ſome writers compared to the figure of a grave matron. The *Ionians* who invented this order, applied it chiefly to decorate their temples: But when applied to the ornamenting a country palace, the rich and extended vale ſeems a proper ſite: Workmen indeed uſe it indifferently in every place.

Palladio gives to the Ionic order the following general proportions.

1. HEIGHT of the column to be 9 modules.

2. THE altitude of the entablature is equal to $\frac{1}{5}$ that of the column, and divided for the architrave, freeze, and cornice, in the proportion of 4, 3, 5.

3. THE height of the pedeſtal equal to 2 modules $37\frac{1}{2}$ minutes; or $\frac{8}{27}$ of the column.

4. HEIGHT of the baſe $\frac{1}{2}$ module; its breadth 1 module $22\frac{1}{2}$ minutes.

5. HEIGHT of the capital and volute is $31\frac{2}{3}$ minutes, and the breadth of its abaco is 1 module $3\frac{1}{3}$ minutes.

6. DIMINUTION of the column is $7\frac{1}{4}$ minutes.

7. IN colonades, the diſtance of the columns in the clear is $2\frac{1}{4}$ modules.

8. IN arches, and the columns ſet on pedeſtals,
   Diſtance of the columns from middle to middle is $7\frac{7}{24}$ modules.
   Height of the arch to its ſoffit is 11 modules.
   Breadth of the pilaſters is $26\frac{1}{2}$ minutes, between the column and arch.

THE diſtance of the modilions in the entablature is 22 minutes, and the breadth of each modilion is 10 minutes; the axis of the column produced always paſſes through the middle of a modilion, which in this order is a plain block repreſenting the end of a beam. The three moſt elegant remains of the ancient Ionic order in *Rome* have their cornice ornamented with dentels inſtead of modilions; and it is the opinion of ſome, eminent for their taſte in Architecture, that in this order dentels would have a better effect than modili-

ons; the heights of theſe dentels were uſually twice their breadth, and their diſtances half their breadth.

The freeze of this order is uſually made ſwelling, and is formed by the ſegment of a circle, whoſe chord is parallel to the axis of the column, and the ſwelling projecting as far as the plateband of the architrave.

The volutes of the capital are now made to project in the directions of the diagonals of the ſquare cap, or abaco, over the volutes, ſo that their drawing ſhould be expreſſed like the volutes in the Roman order: They are much better drawn by an eaſy hand, than by any rules for deſcribing them with the compaſſes, obſerving the limits of their altitude and projection: But the volutes in the ancient examples of this order were curled in a plane parallel to the architrave. Theſe volutes are ſuppoſed to repreſent the plaited treſſes in which the Grecian women uſed to dreſs their hair.

The ſhaft of the column is ſometimes fluted, leaving a fillet or liſt between each channel: In this order there are 24 flutes and fillets.

Of the Corinthian Order.

This order, the moſt elegant of all, is by ſome compared to a very fine woman clad in a wanton ſumptuous habit: It was invented at *Corinth*, and ſoon ſpread into other places to adorn their public buildings. A proper rural ſituation for this order, ſeems to be a ſpot commanding a rich and beautiful proſpect in a fine watered vale.

The general proportions aſſigned by *Palladio* are;

1. The height of the column to be $9\frac{1}{2}$ modules.

2. Height of the entablature equal to $\frac{1}{5}$ that of the column; the architrave, freeze and cornice to be in the proportion of 4, 3, 5; and the projection of the cornice equal to its height.

3. Height

3. HEIGHT of the pedeſtal equal to $\frac{1}{4}$ of the column.

4. THE height of the capital to be $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ of a module; its horns projecting over the bottom of the column $\frac{1}{4}$ of a module.

5. THE height of the baſe equal to $\frac{1}{2}$ module; and its greateſt breadth to be one module and a fifth.

6. THE diminution of the column to be 8 minutes.

7. IN colonades, the intercolumniation is 2 modules.

8. IN arches, and the columns ſet on pedeſtals,

The diſtance of the columns, from middle to middle, to be $6\frac{1}{2}$ modules.

Height of the arch equal to $11\frac{1}{6}$ modules.

Breadth of the pilaſter, between the column and ſides of the paſſage, to be 27 minutes.

IN this order, the ſhaft is frequently cut into 24 flutes, which are ſeparated from one another by as many fillets.

THE capital is compoſed of three tiers of leaves, eight leaves in a tier, with their ſtalks or ſcrols, encircling the body of the capital, which repreſents a baſket, whoſe bottom is juſt as broad as the diameter of the top of the column within the channels: The ornaments of this capital are beſt done by hand, without rule or compaſs, obſerving the proper altitudes and projections of the parts.

THE architrave conſiſts of three faciæs, three fuſaroles, an ogee, and a plateband; the firſt, or lower faciæ projects the ſame as the top of the ſhaft.

THE freeze, which projects the ſame as the top of the ſhaft, has its lower part turned into a kind of cavetto, terminating with the extremity of the plateband of the architrave.

THE breadths of the dentels are $3\frac{1}{3}$ minutes, and their diſtance $1\frac{2}{3}$ minutes.

THE breadths of the modilions are $11\frac{1}{3}$ minutes, and their diſtance in the clear $23\frac{1}{4}$ minutes.

The middle of a dentel ſhould be under the middle of a modilion, and the axis of the column paſſes through the middles of both dentel and modilion.

Of the Composite Order.

This order (the poor invention of the *Romans*, and therefore frequently called the Roman order), is uſually compoſed of the Corinthian and Ionic; the Ionic capital being ſet over the two lower rows of leaves in the Corinthian capital.

Palladio gives us the following general proportions.

1. The height of the column to be 10 modules.
2. The height of the entablature equal to $\frac{1}{5}$ of the column; the architrave, freeze, and cornice, in the proportion of 4, 3, 5; the freeze ſwelling like that of the Ionic.
3. Height of the pedeſtal to be $\frac{1}{3}$ of the column.
4. Height of the capital equal to $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ module, its horns projecting from the center of the column 1 module.
5. Height of the baſe $31\frac{1}{2}$ minutes, and its greateſt breadth $1\frac{1}{5}$ modules.
6. Diminution of the column equal to 8 minutes.
7. In colonades, the intercolumniation is $1\frac{1}{2}$ modules.
8. In arches, and the columns ſet on pedeſtals,
   - Diſtance of the columns from middle to middle is $7\frac{1}{4}$ modules.
   - Height of the arch equal to $12\frac{1}{3}$ modules. In the clear, the height is to the ſpan as 5 to 2.
   - The breadth of the pilaſters between the column and arch is $\frac{7}{10}$ modules, or 42 minutes.

In this order the ſhaft, if fluted, is to have 24 channels and 24 fillets, one between each two flutes.

The volutes of the capital are angular, to have the ſame appearances on every ſide, and they are drawn like thoſe in the Ionic.

The

THE modilions in this order are worked into two faces, with an ogee between them; the breadth of the lower face $9\frac{1}{2}$ minutes, that of the upper $12\frac{1}{2}$; the distance of two modilions at the upper faces is 20 minutes, and at the lower faces 23 minutes; the axis of the column passing through the middle of a modilion.

To draw the Mouldings in Architecture.

THE terminations or ends of flat members, are right lines.

THE astragal, fusarole, and torus, are terminated by a semicircle.

To describe the Torus. Fig. 1. Plate I.

ON AB, its breadth, describe a semicircle.

To make an Ovolo, whose breadth is AB. Fig. 2.

MAKE AC $= \frac{2}{3}$ or $\frac{3}{4}$ of AB, and draw CB.

MAKE the angle CBD equal to the angle BCD.

THEN the intersection of BD with CA will give D the center of the arc BC.

OR, Describe on BC an equilateral triangle; and make the vertex the center.

THE former of these methods is the most graceful.

To make a Cavetto, whose breadth is AB. Fig. 3.

MAKE AC $= \frac{2}{3}$ or $\frac{3}{4}$ of AB; draw BC, and produce the bottom line towards D.

MAKE an angle BCD equal to the angle CBD.

THEN D, the intersection of CD with BD, is the center sought.

OR, On BC describe an equilateral triangle, and the vertex will be the center.

To make a Scotia, whose breadth is AB. Fig. 4.

MAKE AF equal to $\frac{1}{3}$ of AB.

On AF describe the square AC, and on BF describe the square BD.

Then C is the center of the arc EF, and D the center of the arc FG.

To make a Cima, whose breadth is AB. Fig. 5.

Make AC equal to about $\frac{7}{6}$ of AB.

Draw the right line CB, which bisect in D.

On CD and DB, make isosceles triangles, whose legs DE, DF, may be each $\frac{6}{7}$ of the base CD, DB; and the vertexes E and F will be the centers of the arcs CD, DB.

Or, The centers of the arcs CD, DB, may be found by describing equilateral triangles on the right lines CD, DB.

To make a Cymaise, or Ogee, whose breadth is AB. Fig. 6.

Make AC equal to about $\frac{7}{6}$ of AB.

Draw the right line CB, which bisect in D.

Through D draw the right line EF, so, that the angle CDE may be equal to the angle DCE; meeting the upper and lower lines in E and F.

Then E is the center of the arc CD, and F the center of the arc DB.

To describe the curve joining the shaft of a column with its upper or lower fillet, the projection of AB *being given.* Fig. 7.

Make AC equal to twice AB.

Draw CD parallel to AB, and equal to $\frac{5}{4}$ of AC.

Then D is the center of the arc CB.

To draw the gradual diminution of a Column. Fig. 8.

Draw the axis AB of the intended length of the shaft; and parallel thereto, at half a module distance, draw CD; make CE equal to half the proper diminution, and draw EF parallel to BA.

Make

MAKE AG equal to one third of AB; and so high is the shaft to be parallel to its axis; through G draw HI at right angles to AB.

ON HI describe a semicircumference cutting the line EF in the point 4; divide the arc H4 into equal parts at pleasure, suppose 4; and through those points draw the lines 11, 22, 33, 44.

DIVIDE the line GB into a like number of equal parts, as at the points *a*, *b*, *c*; and through these points draw lines parallel to IH; making $aa = 11$, $bb = 22$, $cc = 33$.

THEN a curved line drawn through the extremities H, *a*, *b*, *c*, E, will limit the gradual diminution required.

Palladio describes another method, which is more ready in practice.

LAY a thin ruler by the points D, H, E, and the bending of the ruler will give the gradual diminution required.

To describe the Volute of the Ionic order. Figs. 9, 10.

THE altitude AB, which is $\frac{4}{9}$ of a module, or $26\frac{2}{3}$ minutes, is divided into 8 equal parts, *viz.* 4 from C to A, and 4 from C to B; upon $CD = 3\frac{1}{3}$, one of these parts, a circle is described, and called the eye of the volute, which corresponds with the astragal of the column.

PALLADIO gives the following manner of finding the 12 centers of the volute, which he discovered on an old unfinished capital. Fig. 9.

WITHIN the eye of the volute inscribe a square, whose diagonal is CD; in this square draw the two diameters 13, 24, and these four points 1, 2, 3, 4, are the centers of the arcs A1, 1B, B3, 34, which forms the first revolution.

THE centers of the arcs forming the second and third revolutions are thus found; see the eye of the volute drawn at large. Fig. 9.

DIVIDE

Divide the radii o1, o2, o3, o4, each into 3 equal parts, as at the points 5, 6, 7, 8, 9, 10, 11, 12, and these will be the centers of the remaining arcs, the last of which is to coincide with the point c, in the eye.

Goldman observing that in this construction the ends and beginnings of the arcs were not at right angles to the same radii, contrived the following construction. See Fig. 10. and its eye drawn at large.

Upon one half of cd, describe the square 1, 2, 3, 4; and draw the lines o2, o3; divide o1, o4, each into 3 equal parts; then lines drawn through those points parallel to 1, 2, their intersections with 14, o2, o3, will be centers of the volute.

So the points, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, will be the centers of the twelve arcs which together form the outward curve of the volute.

In either method, the centers of the inner curve may be thus found.

Take o*a* equal to $\frac{7}{8}$ of o1; divide o*a* into three equal parts, and these divisions will give centers of the inner curve; the two eyes drawn at large will shew how the 12 inner centers are found, where they are distinguished by large points; the 12 centers of the outward curve being marked by the figures.

In the describing of these volutes, it will frequently happen, that the last quadrant will not fall on its true termination, occasioned by the radii of the several quadrants not being exactly taken by the compasses: In order to avoid this inaccuracy, at least in some degree, here is subjoined a table shewing the length of each radius, computed from *Goldman*'s method: But it may also be applied to *Palladio*'s, the radius of the largest quadrant not differing $\frac{3}{100}$ of a minute, or $\frac{1}{2000}$ of a module from the truth; and excepting the arc described from the first center, the rest may be made quadrants in the same manner as shewn in *Goldman*'s method.

A TABLE *of the lengths, in minutes, of the ſeveral radii of the outward and inner volutes.*

Nº Rad.	Outward Curve.	Inward Curve.	In parts of 1ſt rad. Outward.	Inward.
1	$\frac{85}{6}=14,166$	$\frac{605}{48}=12,604$	100,000	88,969
2	$\frac{75}{6}=12,500$	$\frac{535}{48}=11,146$	88,235	78,677
3	$\frac{65}{6}=10,833$	$\frac{465}{48}=9,687$	76,468	68,379
4	$\frac{55}{6}=9,166$	$\frac{395}{48}=8,229$	64,705	58,087
5	$\frac{70}{9}=7,777$	$\frac{1010}{144}=7,014$	54,901	49,510
6	$\frac{60}{9}=6,666$	$\frac{870}{144}=6,041$	47,058	42,642
7	$\frac{50}{9}=5,555$	$\frac{730}{144}=5,069$	39,215	35,781
8	$\frac{40}{9}=4,444$	$\frac{590}{144}=4,097$	31,372	28,920
9	$\frac{65}{18}=3,611$	$\frac{485}{144}=3,368$	25,490	23,774
10	$\frac{55}{18}=3,055$	$\frac{415}{144}=2,882$	21,568	20,343
11	$\frac{45}{18}=2,500$	$\frac{345}{144}=2,395$	17,647	16,906
12	$\frac{35}{18}=1,944$	$\frac{275}{144}=1,909$	13,725	13,475

To uſe this table, a ſcale of $\frac{1}{4}$ of a module ſhould be made, and divided into 15 minutes, and the ex-tream diviſion decimally divided, whereby the lengths of the ſeveral radii may be taken: But as the ſector is an univerſal ſcale, there are two other columns add-ed,

ed, applicable to the sector; where the longer radius 14,166 is made a transverse distance to 10 and 10, or 100 and 100, on the line of lines, and all the other radii of both curves are proportioned thereto: Now the centers of the curves being found as shewn in the eyes of the volute, the several radii may be taken from the sector, and the curves more accurately described than by any other method.

To describe the Flutings and Fillets in channelled columns. Fig. 11.

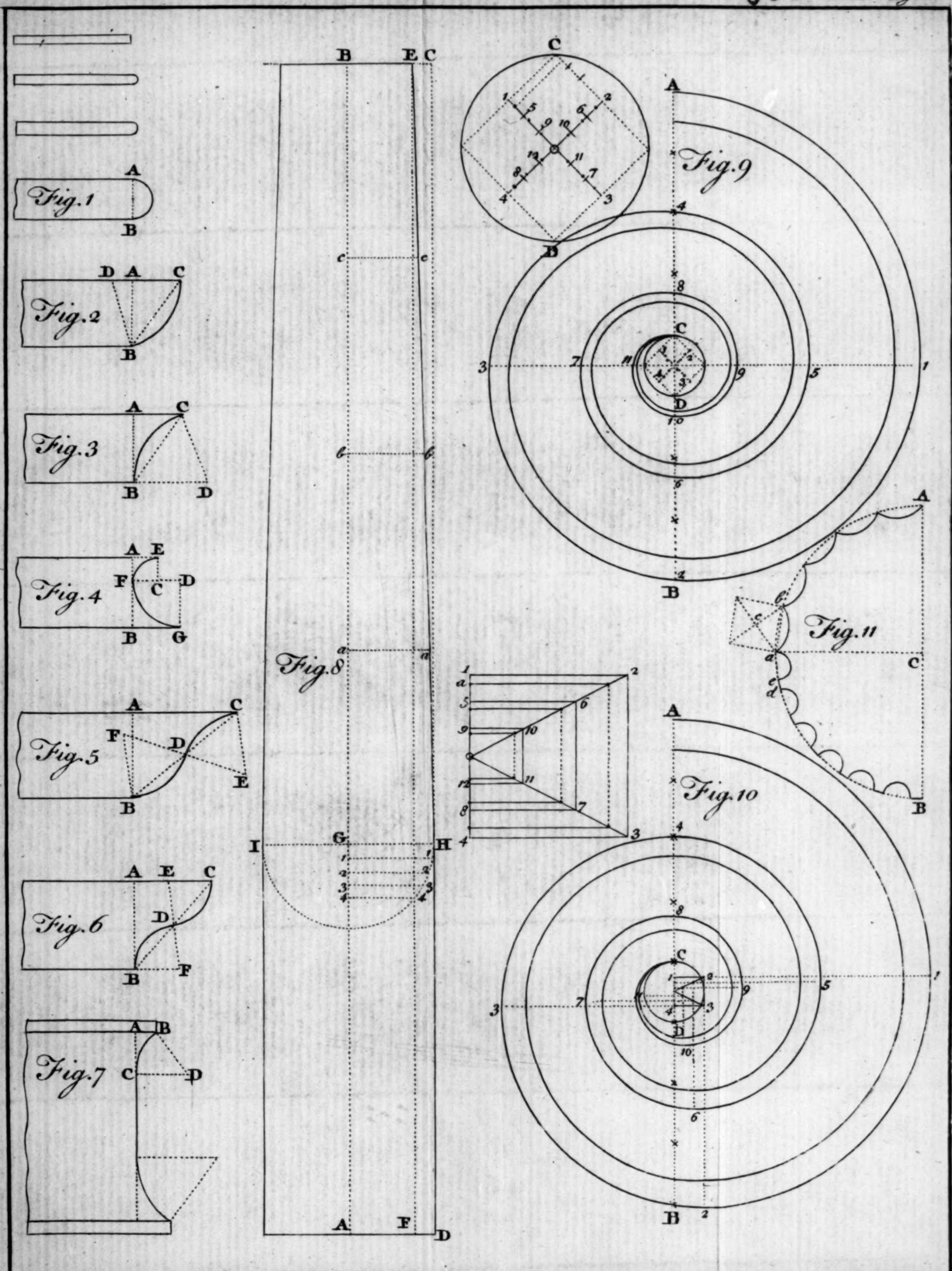
In the Doric, the circumference of the column being divided into 20 equal parts (here the $\frac{1}{4}$ circumference is divided into 5), of which *ab* is one; on *ab* describe a square, and the center *c* of that square is the center of the channel or flute required.

In the Ionic, and Corinthian, divide the circumference of the column into 24 equal parts (here the $\frac{1}{4}$ circumference is divided into 6), of which *ad* is one; divide *ad* into 4 equal parts; then $ae = \frac{3}{4}ad$ is the breadth of the flute, and $ed = \frac{1}{4}ad$ is the breadth of the fillet.

The flutes are semicircles described on the chords of their arcs in the column.

In the three following tables are contained the heights and projections of the parts of each order, according to the proportions given by *Palladio*; the orders of this architect were chosen, because the *English*, at present, are more fond of copying his productions, than those of any other architect.

The first table serves for the pedestal, the second for the column, and the third for the entablature, of each order. Each table is divided into seven principal columns: In the first, beginning at the left hand, is contained the names of the primary divisions; in the second those of the several divisions and members in the orders; and the other five, titled with *Tuscan*, *Doric*



B. Cole Sculp.

Doric, Ionic, Corinthian, Roman, contain the numbers expressing the altitudes, and projections taken from the axis, or middle of the column, of the several members belonging to their corresponding orders.

The column containing each order, is divided, first into two other columns, one shewing the altitudes, and signed Alt. and the other, the projections, and signed Proj. Each of these is also divided into two other columns, one containing modules, and marked Mo. and the other, the minutes and parts, and marked Mi.

Under the table of the pedestal there is another table, shewing the general proportions for the heights of the orders.

In each of the orders of architecture, the height of the order, and the diameter of the column, have a constant relation to one another.

Therefore, if the diameter of the column be given, the height of the order is given also: And having determined by what scale the order is to be drawn, such as $\frac{1}{2}$ inch, 1 inch, 2 inches, &c. to a foot or yard, &c. Take from such scale, the part or parts expressing the diameter of the column, and make this extent a transverse distance to 6 and 6 (*i. e.* 60 and 60) on the scales of lines, and the sector will be opened so, that the several proportions of the order may be taken from it.

Exam. *Suppose the diameter of a column is to be* 18 *inches; and the drawing of the order is to be delineated from a scale of an inch to a foot: that is, the diameter of the column in the drawing is to be an inch and half.*

Make the transverse distance of 6 and 6, on the scales of lines, equal to $1\frac{1}{2}$ inch, and the sector is fitted for the scale.

If the height of the order is given, divide this height, by the height of the order in the table; and the quotient will be the diameter of the column.

 Exam.

Exam. *What must be the diameter of the column in the* Ionic *order, when the whole height of the order is fixed at* 18 *feet* 6 *inches.*

The height of the order in the table is 13 mo. $29\frac{1}{4}$ mi. $= 13\frac{29,25}{60} = 13,4875$ modules: And 18 f. 6 in. $= 18,5$ feet. Therefore $\frac{18,5}{13,4875} = 1,3709$ feet $= 1$ f. $4\frac{1}{2}$ inches nearly: And the sector may be fitted to this, as before directed, according to the intended size of the draught.

To delineate an Order by these Tables.

Having determined the diameter of the column at bottom, and set the sector to the intended scale, draw a line to represent the axis or middle of the order.

On this line, lay the parts for the heights of the pedestal, column, and entablature, taken from the table of general proportions.

Within each of these parts respectively, lay the several altitudes taken from the tables of particulars, under the word Alt. Through each of the points marked on the axis, draw lines perpendicular to the axis, or draw one line perpendicular, and the others parallel thereto.

On the lines drawn perpendicular to the axis, lay the projections corresponding to the respective altitudes; these projections are to be laid on both sides of the axis, for the pedestal and column; and only on one side, for the entablature, join the extremities of the projections with such lines as are proper to express the respective mouldings and parts: And the order, exclusive of its ornaments, will be delineated.

As the altitudes of many of the parts are very small, it will not be convenient, if possible, to take from the scale of lines, such small parts alone; therefore it may be

be best to proceed as in the following example of the *Ionic* order.

To construct the Pedestal. Plate II.

In the line AD, which represents the axis of the order, take the base $Aa = 42\frac{1}{2}$ min., the die $ad =$ 1 mod. K 35 min.; and the capital $dD = 22\frac{3}{4}$ min. Then to draw the small members in the base and cornice, proceed thus.

To the minutes in the base, $42\frac{1}{2}$, add some even number of minutes, suppose $30 = aB$, and the sum $72\frac{1}{2}$ is equal to AB; then compose a table, such as the following one, wherein the alt. of the plinth is subtracted out of the No. $72\frac{1}{2}$; then the torus out of this remainder; then the cyma out of this remainder; then the fillet out of this; and lastly, the cavetto out of this remainder. Thus,

			Min.
Base with 30 minutes		. . . .	$72\frac{1}{2}$
This less by the plinth,	$28\frac{1}{2}$,	remains . . .	44
This less by the torus,	4,	remains . . .	40
This less by the fillet,	$0\frac{3}{4}$,	remains . .	$39\frac{1}{4}$
This less by the cyma,	5,	remains . . .	$34\frac{1}{4}$
This less by the fillet,	$0\frac{3}{4}$,	remains . .	$33\frac{1}{2}$
This less by the cavetto,	$3\frac{1}{2}$,	remains . . .	30,

the minutes first added.

Then the several numbers in the table may be taken from the line of lines on the sector, and applied from B towards A. Thus,

Make $B1 = 44$, $B2 = 40$, $B3 = 39\frac{1}{4}$, $B4 = 34\frac{1}{4}$, $B3 = 33\frac{1}{4}$; draw lines through these points at right angles to AD, and on these lines lay the respective projections, as shewn in the general table; then the proper curvature or figure being drawn at the extremities of the numbers, the base of the pedestal will be made.

It will be found most convenient to lay off the numbers from the greater to the lesser ones; for then there is only one motion required in the joints of the com-

compaſſes, which is, to bring them cloſer and cloſer every diſtance laid down.

AND in the ſame manner, for the cornice of the pedeſtal, take a point C, 30 minutes below the cornice; and tabulate as before.

Cornice with 30 min.				$52\frac{3}{4}$	= CD
This leſs by the fillet or cap,		$2\frac{1}{2}$,	leaves	$50\frac{1}{4}$	= C1
Ditto	ogee . . .	$3\frac{1}{2}$,	ditto	$46\frac{3}{4}$	= C2
Ditto	corona . .	$4\frac{1}{2}$	. . .	$42\frac{1}{4}$	= C3
Ditto	fillet . . .	$1\frac{3}{4}$	. . .	$40\frac{1}{2}$	= C4
Ditto	cyma . . .	$5\frac{1}{4}$	. . .	$35\frac{1}{4}$	= C5
Ditto	fillet . . .	$1\frac{3}{4}$	. . .	$33\frac{1}{2}$	= C6
Ditto	cavetto . .	$3\frac{1}{2}$	. . .	30	= C*d*.

THESE numbers laid from C towards D, gives the altitudes of the members of the cornice.

IN like manner the mouldings about the baſe and capital are laid down, by taking 30 minutes in the ſhaft both above the baſe and below the capital; having firſt ſet on the axis, the reſpective heights of the baſe, ſhaft, and capital,

Thus for the Baſe.

THE baſe $33\frac{1}{2}$ min. with 30 added		=	$63\frac{1}{2}$	= SD
This leſs by the plinth 10 min. leaves			$53\frac{1}{2}$	= S1
Ditto	torus	$7\frac{1}{2}$	46	= S2
Ditto	fillet	$1\frac{1}{4}$	$44\frac{3}{4}$	= S3
Ditto	ſcotia	$4\frac{2}{3}$	$40\frac{1}{12}$	= S4
Ditto	fillet	$1\frac{1}{4}$	$38\frac{5}{6}$	= S5
Ditto	torus	$5\frac{1}{3}$	$33\frac{1}{2}$	= S6
Ditto	aſtragal	$2\frac{1}{4}$	$31\frac{1}{4}$	= S7
Ditto	fillet	$1\frac{1}{4}$	30	= S8.

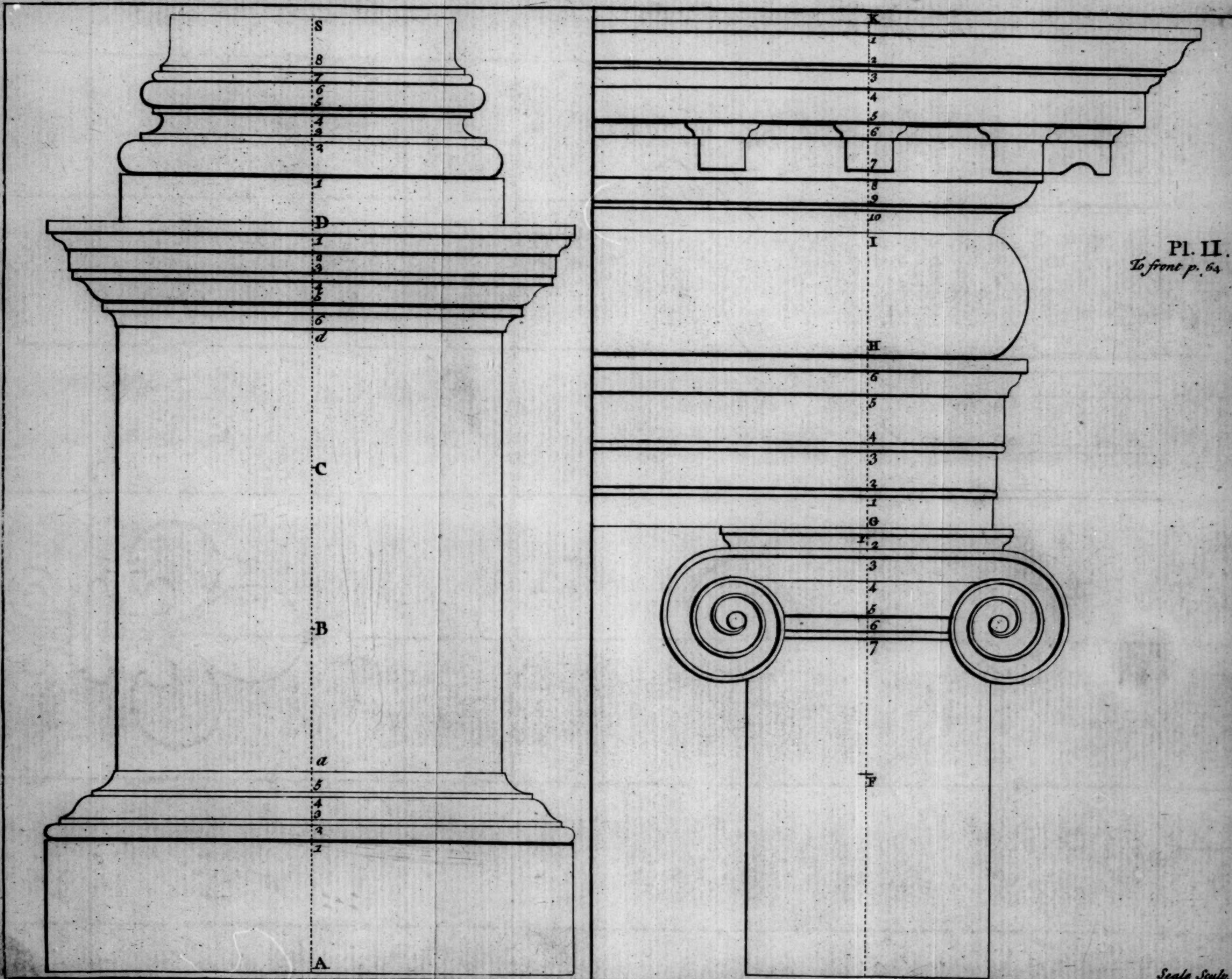
S8 is here ſuppoſed to be 30, though the plate is not high enough to admit 30 minutes to be laid in the ſhaft of the column.

For the Capital.

THE capital $24\frac{1}{4}$ with 30 added, gives			$54\frac{1}{4}$	= FG
This leſs by the plateband		$1\frac{3}{4}$, leaves	$52\frac{1}{2}$	= F1
Ditto	ogee . .	$3\frac{1}{3}$ . . .	$49\frac{1}{6}$	= F2

This

64



Pl. II.
To front p. 64

This less by the rim of volute		$1\frac{1}{3}$,	leaves	$47\frac{5}{6}$	= F3
Ditto	hollow	$5\frac{1}{3}$	. . .	$42\frac{1}{2}$	= F4
Ditto	ovolo . .	$7\frac{1}{2}$	. . .	35	= F5
Ditto	astragal	$3\frac{1}{3}$	. . .	$31\frac{2}{3}$	= F6
Ditto	fillet . .	$1\frac{2}{3}$	. . .	30	= F7.

To construct the Cornice.

In the axis take GH = 36 for the architrave, HI = 27 for the freeze, and IK = 46 for the cornice. Then,

For the parts of the Architrave.

To the freeze 27 add	HG	36,	gives	63	= IG
This less by the	first face	$6\frac{1}{2}$,	leaves	$56\frac{1}{2}$	= I1
Ditto	fusarole	$1\frac{1}{4}$	. . .	$55\frac{1}{4}$	= I2
Ditto	2d face .	$8\frac{1}{3}$	. . .	$46\frac{11}{12}$	= I3
Ditto	fusarole	2	. . .	$44\frac{11}{12}$	= I4
Ditto	3d face	$10\frac{1}{2}$	. . .	$34\frac{5}{12}$	= I5
Ditto	ogee . .	$4\frac{3}{4}$	. . .	$29\frac{2}{3}$	= I6
Ditto	fillet . .	$2\frac{2}{3}$	. . .	27	= IH.

For the Cornice.

To the freeze 27 add the cornice		46,	gives	73	= HK
This less by the	fillet . .	$2\frac{1}{2}$,	leaves	$70\frac{1}{2}$	= H1
Ditto	cima .	7	. . . .	$63\frac{1}{2}$	= H2
Ditto	fillet . .	1	. . . .	$62\frac{1}{2}$	= H3
Ditto	ogee . .	$3\frac{1}{2}$	. . . .	59	= H4
Ditto	corona	8	. . . .	51	= H5
Ditto	ogee . .	3	. . . .	48	= H6
Ditto	modilion	$7\frac{1}{2}$	. . . .	$40\frac{1}{2}$	= H7
Ditto	fillet . .	$1\frac{1}{2}$	. . . .	39	= H8
Ditto	ovolo . .	6	. . . .	33	= H9
Ditto	fillet . .	1	. . . . .	32	= H10
Ditto	cavetto .	5	. . . . .	27	= HI.

Tables may be made in like manner for either of the orders, to be taken from the sector: The projections from the axis being all of them large numbers, they may be taken from the sector easily enough after it is set to the diameter of the column, as before shewn.

A little reflection will make this very clear, and perhaps more so, than by bestowing more words thereon.

A TABLE *shewing the Altitudes and Projections of*
Order; according to the

	Names of the Members.	Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet	—	—	—	—	o	$3\frac{2}{3}$	o	56
	Ogee	—	—	—	—	—	—	—	—
	Corona	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Cima	—	—	—	—	o	9	—	—
	Fillet	—	—	—	—	o	$\begin{cases}1\frac{1}{4}\\1\frac{1}{4}\end{cases}$	o	$\begin{cases}47\\45\frac{3}{4}\end{cases}$
	Astragal	—	—	—	—	—	—	—	—
	Ogee	—	—	—	—	—	—	—	—
	Cavetto	—	—	—	—	o	5	o	$41\frac{1}{4}$
	Fillet	—	—	—	—	—	—	—	—
	The Cornice	—	—	—	—	o	$26\frac{1}{6}$	—	—
	THE DIE	1	o	o	42	1	20	o	40
	The Base	—	—	—	—	o	40	—	—
BASE.	Fillet	—	—	—	—	—	—	—	—
	Cavetto	—	—	—	—	o	5	o	$41\frac{1}{4}$
	Ogee	—	—	—	—	—	—	—	—
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	o	$\begin{cases}1\frac{1}{4}\\1\frac{1}{4}\end{cases}$	o	$\begin{cases}46\\47\frac{1}{4}\end{cases}$
	Cima	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Torus	—	—	—	—	—	5	o	50
	Plinth	—	—	—	—	—	$27\frac{1}{2}$	o	50

A TABLE *of general*

The Order	9	$44\frac{1}{2}$	—	—	12	$12\frac{1}{6}$	—	—
The Entablature	1	$44\frac{1}{2}$	—	—	1	53	—	—
The Column	7	o	—	—	8	o	—	—
The Pedestal	1	o	—	—	2	$20\frac{1}{6}$	—	—

every Moulding and Part in the Pedestals of each Proportions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
o	2½	o	56¼	o	2½	o	57	o	2½	o	57
o	3½	o	{ 55¼ 53¼	o	3½	o	{ 56 54¼	o	3½	o	{ 56 54¼
o	4½	o	52¼	o	4¼	o	53¾	o	5½	o	53½
o	1¾	o	51¾	—	—	—	—	o	1	o	52¾
o	5¼	—	—	o	4¼	o	{ 49¼ 46	o	8½	—	—
o	1¾	o	44¾	o	0¾	o	46	—	—	—	—
—	—	—	—	—	—	—	—	o	3	o	46¼
—	—	—	—	o	3¾	o	{ 45 43	—	—	—	—
o	3½	o	41¾	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	o	1¾	o	44¾
o	22¾	—	—	o	19	—	—	o	25¼	—	—
1	35	o	41¼	1	36	o	42	2	6½	o	42
o	42½	—	—	o	38	—	—	o	50	—	—
—	—	—	—	—	—	—	—	o	1	o	45½
o	3½	o	41¾	—	—	—	—	—	—	—	—
—	—	—	—	o	4	o	{ 43 46	—	—		—
—	—	—	—	—	—	—	—	o	3	o	47
o	0¾	o	47¼	o	0¾	o	47	—	—	—	—
o	5	—	—	o	5	—	—	o	7½	o	{ 45¼ 54¾
o	0¾	o	53¾	o	0¾	o	55	o	1	o	54¾
o	4	o	56¼	o	4	o	57	o	4½	o	57
o	28½	o	56¼	o	23½	o	57	o	23	o	57

Proportions for the Orders.

13	29¼		—	13	7	—	—	1	22¼	—	—
1	49	—	—	1	54	—	—	2	o	—	—
9	o	—	—	9	30	—	—	10	o	—	—
2	40¼	—	—	2	33	—	—	3	22¼	—	—

A TABLE, *shewing the Altitudes and Projections of according to the Propor-*

Names of the Members.		Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CAPITAL.	Angular Volutes	—	—	—	—	—	—	—	—
	Abacus: Ovolo	—	—	—	—	—	—	—	—
	Abacus: Fillet	—	—	—	—	0	1¼	0	38¾
	Abacus: Cavetto	—	—	—	—	—	—	—	—
	Basket Rim	—	—	—	—	—	—	—	—
	Ogee	—	—	—	—	0	2½	0	37¾ / 36¼
	Abacus	0	10	0	30	0	6¼	0	35¾
	Volute: fillet or rim	—	—	—	—	—	—	—	—
	Volute: chan. or hollow	—	—	—	—	—	—	—	—
	Ovolo	0	10	0	29	0	6½	0	34½
	Astragal	—	—	—	—	—	—	—	—
	Fillet	0	1½	0	24½	0	1 1/7	0	29¾
						0	1 1/8	0	28½
						0	1 1/9	0	27¼
	Collarino	0	8½	0	22½	0	10	0	26
	Middle Volute	—	—	—	—	—	—	—	—
	Cours. of leaves, folding half their height: 3d	—	—	—	—	—	—	—	—
	2d	—	—	—	—	—	—	—	—
	1st	—	—	—	—	—	—	—	—
SHAFT.	Astragal	0	4	0	27	0	3½	0	30
	Fillet	0	1½	0	24½	0	1½	0	28¼
	Body of the Column	5	54½	0	22½ / 30	6	53¾	0	26 / 30
	Fillet	0	2½	0	33¾	0	1¼	0	33½
	Astragal	—	—	—	—	—	—	—	—
BASE.	Torus	—	—	—	—	0	5½	0	36⅔
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	0	1¼	0	35
	Scotia	—	—	—	—	0	4½	0	33⅓
	Fillet	—	—	—	—	0	1¼	0	36⅔
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Scotia	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Torus	0	12½	0	40	0	7½	0	40
	Plinth	0	15	0	40	0	10	0	40
	Base	0	22½	—	—	0	30	—	—
	Shaft	6	2½	—	—	7	0	—	—
	Capital	0	30	—	—	0	30	—	—

SECOND.

every Moulding and Part in the Columns of each Order ;
tions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
0	26⅔	0	41⅔	0	12	0	41	0	25⅔	0	35
—	—	—	—	0	3	0	45	0	3	0	44
0	1¾	0	31½	0	1⅓	0	42	0	1⅓	0	42½
—	—	—	—	0	5⅔	0	39	0	5⅔	0	41
—	—	—	—	0	2½	—	—	—	—	—	—
0	3⅓	0	30¾ 29	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
0	1⅓	—	—	—	—	—	—	—	—	—	—
0	5⅓	—	—	—	—	—	—	—	—	—	—
0	7½	0	35	—	—	—	—	0	5½	0	32
—	—	—	—	—	—	—	—	0	3	0	26
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	0	1½	0	24
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	0	9½	—	—	—	—	—	—
—	—	—	—	0	8	—	—	0	10	—	—
—	—	—	—	0	20	0	41	0	20	0	39
—	—	—	—	0	20	0	35	0	20	0	35
0	3⅓	0	30	0	3⅔	0	30	0	4	0	30
0	1⅔	0	28⅓	0	1⅓	0	28	0	1½	0	28
8	2¼	0	26 30	7	40¾	0	26 30	8	9	0	26 30
0	1¼	0	33	0	1¾	0	33½	0	1	0	34
0	2¼	0	34½	0	2½	0	35½	0	3	0	35½
0	5⅓	0	37	0	5	0	37½	0	4½	0	37
—	—	—	—	0	1½	0	35½	—	—	—	—
0	1¼	0	34½	0	0⅔	0	34	0	0⅔	0	35½
0	4⅔	—	—	0	3¾	—	—	0	3	—	—
0	1¼	0	37	0	0¾	0	37	0	0½	0	36½
—	—	—	—	0	1¾	0	38½	0	1	0	37
								0	1	0	37
—	—	—	—	—	—	—	—	0	0½	0	36½
—	—	—	—	—	—	—	—	0	3	—	—
—	—	—	—	—	—	—	—	0	0⅔	0	38½
0	7½	0	41¼	0	7	0	42	0	7	0	42
0	10	0	41¼	0	9⅔	0	42	0	9⅔	0	42
0	30	—	—	0	30	—	—	0	31½	—	—
8	10½	—	—	7	5	—	—	8	18½	—	—
0	19¼	—	—	1	10	—	—	1	10	—	—

A TABLE, *shewing the Altitudes and Projections of* … *Order; according to the Pro-*

	Names of the Members.	Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet	0	$3\frac{1}{2}$	1	6	0	$2\frac{1}{4}$	1	16
	Cima	0	10	—	—	0	$6\frac{1}{4}$	—	—
	Fillet	0	2	0	$54\frac{1}{4}$	0	$0\frac{3}{4}$	1	8
	Ogee	—	—	—	—	0	$3\frac{1}{4}$	1 1	7 $5\frac{1}{2}$
	Corona	0	10	0	$52\frac{1}{4}$	0	8	1	$4\frac{1}{2}$
	Ovolo	0	9	0	42	0	6	0	$39\frac{1}{2}$
	Fillet or Astragal	0	$1\frac{1}{2}$	0	32	0	1	0	$35\frac{1}{2}$
	Ogee	—	—	—	—	—	—	—	—
	Modilion 2d Face	—	—	—	—	—	—	—	—
	Modilion Ogeee	—	—	—	—	—	—	—	—
	Modilion 1st Face	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Ovolo	—	—	—	—	—	—	—	—
	Ogee	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Dentel	—	—	—	—	—	—	—	—
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Ogee	—	—	—	—	—	—	—	—
	Cavetto	0	$7\frac{1}{2}$	0	$23\frac{1}{2}$	0	5	0	31
	Trigliphs Capital	—	—	—	—	0	5	0	$30\frac{1}{2}$
	The Cornice	0	$43\frac{1}{2}$	—	—	0	38	—	—
	THE FREEZE	0	20	0	$22\frac{1}{2}$	0	45	0	26
	The Architrave	0	35	—	—	0	30	—	—
ARCHITRAVE.	Fillet	0	5	0	$27\frac{1}{2}$	0	$4\frac{1}{2}$	0	28
	Cavetto	—	—	—	—	—	—	—	—
	Ogee	—	—	—	—	—	—	—	—
	Astragal or Fusarole	—	—	—	—	—	—	—	—
	Third Face	—	—	—	—	—	—	—	—
	Astragal or Fusarole	—	—	—	—	—	—	—	—
	Second Face	0	$17\frac{1}{2}$	0	24	0	$14\frac{1}{2}$	0	27
	Ogee	—	—	—	—	—	—	—	—
	Astragal or Fusarole	—	—	—	—	—	—	—	—
	First Face	0	$12\frac{1}{2}$	0	$22\frac{1}{2}$	0	11	26	26

THIRD.

every Moulding and Part in the Entablature of each portions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo	Mi.	Mo.	Mi.	Mo.	Mi.
0	2½	1	1½	0	2¼	1	14	0	2½	1	18½
0	7	—	—	0	6¼	—	—	0	8	—	—
0	1	1	4	0	0⅓	1	6½	0	1	1	10
0	3½	1 0	3 0½	0	3	1 1	5½ 4	0	3¾	1 1	9 6
0	8	0	59½	0	7⅓	1	3	0	9½	1	5
—	—	—	—	—	—	—	—	0	2½	0	55
—	—	—	—	0	0⅔	1	2	0	1¼	0	54
0	3	0	55 53	0	2⅓	1 0	1 59	—	—	—	—
	—	—	—	—	—	—	—	0	6½	0	53
0	7½	0	52	0	7¼	0	40¼	0	1¾	0	52½
	—	—	—	—	—	—	—	0	3¼	0	51
0	1½	0	37	0	1	0	40	0	1	0	51
0	6	0	36	0	4½	0	39	—	—	—	—
—	—	—	—	—	—	—	—	0	5	0	35½ 29
—	—	—	—	0	1	0	36	—	—	—	—
—	—	—	—	0	5½	0	35	—	—	—	—
—	—	—	—	—	—	—	—	0	2	0	30
0	1	0	31½	0	1	0	32	0	2	0	28½
—	—	—	—	0	4½	0	31 27	—	—	—	—
0	5	0	27	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
0	46	—	—	0	47½	—	—	0	50	—	—
0	27	0	34	0	28½	0	26	0	30	0	35
0	3	—	—	0	38	—	—	0	40	—	—
0	2⅔	0	34	0	2½	0	34½	0	2⅛	0	35
—	—	—	—	—	—	—	—	0	4⅛	0	32
0	4¼	0	33 30	0	5	0	33½ 30	0	3⅔	0	31 29
—	—	—	—	0	2	0	29½	—	—	—	—
0	10½	0	29	0	10½	0	28	—	—	—	—
0	2	0	29	0	1¾	0	28	0	1 5/12	—	29
0	8⅓	0	27½	0	8¼	0	27	0	15	0	28
—	—	—	—	—	—	—	—	0	2⅔	0	27½ 26½
0	1¼	0	27½	0	1¾	0	27	—	—	—	—
0	6½	0	26½	0	6¼	0	26	0	11	0	26

Sect. XIII.

Some Uses of the Scales of Polygons. Pl. VI.

PROBLEM XVI.

In a given circle, whose diameter is AB, *to inscribe a regular octagon.* Fig. 22.

Solution. Open the legs of the sector, till the transverse distance of 6 and 6, be equal to AB: Then will the transverse distance of 8 and 8, be the side of an octagon which will be inscribed in the given circle.

In like manner may any other polygon not exceeding 12 sides, be inscribed in a given circle.

PROBLEM XVII.

On a given line AB, *to describe a regular pentagon.* Fig. 23.

Solution. 1st. Make AB a transverse distance to 5 and 5.

2d. At that opening of the sector, take the transverse distance of 6 and 6; and with this radius, on the points A, B, as centers, describe arcs cutting in C.

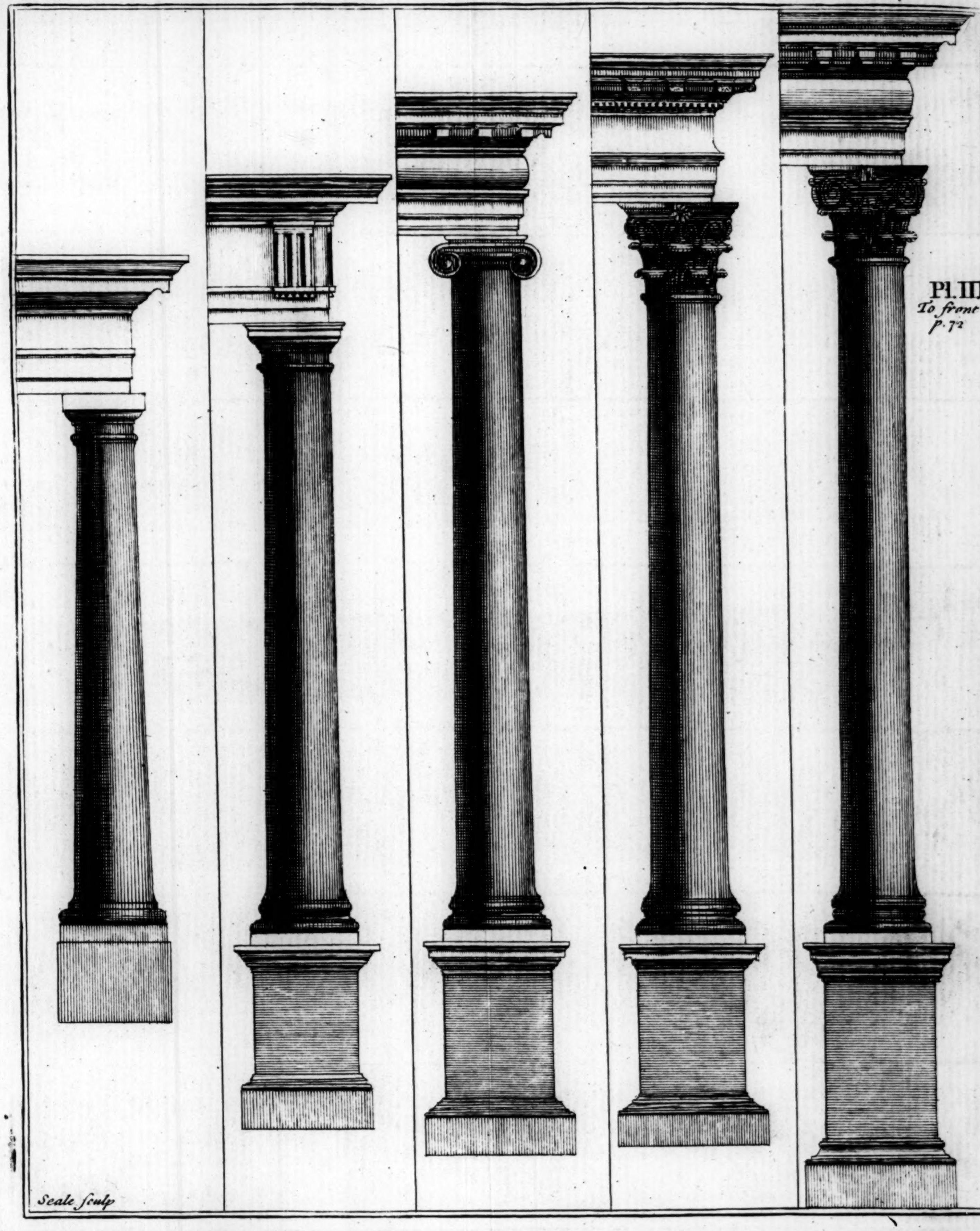
3d. On C as a center, with the same radius, describe a circumference passing through the points A, B; and in this circle may the pentagon, whose side is AB, be inscribed.

By a like process may any other polygon, of not more than 12 sides, be described on a given line.

The scales of chords will solve these two problems, or any other of the like kind: Thus,

In a circle whose diameter is AB, *to describe a regular polygon of* 24 *sides.* Fig. 24.

Solution. 1st. Make the diameter AB, a transverse distance to 60 and 60, on the scales of chords.



Pl. III
To front p. 72

2d. DIVIDE 360 by 24; the quotient gives 15.

3d. TAKE the transverse distance of 15 and 15, and this will be the chord of the 24th part of the circumference.

As there are great difficulties attending the taking of divisions accurately from scales; therefore in this problem, where a distance is to be repeated several times, it will be best to proceed thus.

WITH the chord of 60 degrees, divide the circumference into six equal parts.

IN every division of 60 degrees, lay down, 1st. The chord of 15 degrees. 2d. The chord of 30 degrees. 3d. The chord of 45 degrees, beginning always at the same point.

IF methods like this be pursued in all similar cases, the error in taking distances, will not be multiplied into any of the divisions following the first.

SECT. XIV.

Some Uses of the Scales of Chords.

THESE double scales of chords, are more convenient than the single scales, such as described on the plain scale; for on the sector, the radius with which the arc is to be described, may be of any length between the transverse distance of 60 and 60, when the legs are close, and that of the transverse distance of 60 and 60, when the legs are opened as far as the instrument will admit of. But with the chords on the plain scale, the arc described, must be always of the same radius.

PROBLEM

PROBLEM XVIII.

To protract, or lay down, a right lined angle, BAC, *which shall contain a given number of degrees,* Pl. VI.

CASE I. *When the degrees given are under* 60 : *Suppose* 46. Fig. 25.

1st. AT any opening of the sector, take the transverse distance of 60 and 60, (on the chords;) and with this opening, describe an arc BC.

2d. TAKE the transverse distance of the given degrees 46, and lay this distance on the arc from any point B, to C; marking the extremities B, C, of the said distance.

3d. FROM the center A of the arc, draw two lines AC, AB, each passing through one extremity of the distance BC, laid on the arc; and these two lines will contain the angle required.

CASE II. *When the degrees given are more than* 60 : *Suppose* 148.

1st. DESCRIBE the arc BC as before.

2d. TAKE the transverse distance of $\frac{1}{2}$ or $\frac{1}{3}$, of the given degrees 148; suppose $\frac{1}{3} = 49\frac{1}{3}$ degrees; lay this distance on the arc thrice; *viz.* from B to *a*, from *a* to *b*, from *b* to D.

3d. FROM the center A, draw two lines AB, AD; and the angle BAD will contain the degrees required.

When an angle containing less than 5 *degrees, suppose* $3\frac{1}{2}$, *is to be made, it is most convenient to proceed thus.*

1st. DESCRIBE the arch DG with the chord of 60 degrees.

2d. FROM some point D, lay the chord of 60 degrees to G; and the chord of $56\frac{1}{2}$ degrees ($= 60^\circ - 3\frac{1}{2}^\circ$) from D to E.

3d. LINES drawn from the center A, through G and E, will form the angle AGE, of $3\frac{1}{2}$ degrees.

IF

If the radius of the arc or circle is to be of a given length; then make the transverse distance of 60 and 60, equal to that assigned length.

Either of these scales of chords, may be used singly in the manner directed in the use of chords on the plane scale.

From what has been said about the protracting of an angle to contain a given number of degrees, it will be easy to see how to find the degrees which are contained in a given angle already laid down.

PROBLEM XIX.

To delineate the visual lines of a survey; by having given, the bearings and distances from each other, of the stations terminating those visual lines.

Exam. Suppose in the field-book of a survey, the bearings and distances of the stations were expressed as follows:

⊙ signifies Station.
B ——— Bearing.
D ——— Distance.

	⊙ 1.	B 70° 50′	D 1080 links.
	⊙ 2.	B 128 10	D 580.
	⊙ 3.	B 32 15	D 605.
	⊙ 4.	B 287 30	D 766.
	⊙ 5.	B 50 45	D 940.
	⊙ 6.	B 73 55	D 1085.
	⊙ 7.	B 183 25	D 700.
Return to D 314 in ⊙ 7.	⊙ 8.	B 133 30	D 510 to ⊙ 5.
	⊙ 9.	B 186 30	D 390 to ⊙ 2.
Return to D 700 in ⊙ 7.	⊙ 10.	B 209 20	D 668 cutting [1st D.
Return to ⊙ 10.	⊙ 11.	B 275 30	D 800.
	⊙ 12.	B 171 50	D 784 to ⊙ 1.

The bearings are counted from the North, Eastward. Therefore all the bearings under 90 degrees, fall between the N. and E. or in the 1ſt quadrant.

Bearings between 90° and 180°, fall between the E. and S. or in the 2d quadrant.

Those between 180° and 270°, fall between the S. and W. or in the 3d quadrant.

And thoſe between 270° and 360°, fall between the W. and N. or in the 4th quadrant.

Solution. 1ſt. Take from the chords the tranſverſe diſtance of 60 and 60, (the ſector being opened at pleaſure,) with this radius deſcribe a circumference, and draw the diameters NS. WE. at right angles. Pl. VI. Fig. 31.

2d. The firſt bearing 70°. 50′ is in the firſt quadrant, but being more than 60°, take the tranſverſe diſtance of the half of 70° 50′, and apply this extent in the circumference twice from N. towards E, and the point correſponding to the 1ſt bearing will be obtained, which mark with the figure 1.

3d. The ſecond bearing 128° 10′, falls in the ſecond quadrant; its ſupplement to 180° is 51° 50′, that is 51° 50′ from the S. point. Now take the tranſverſe diſtance of 51° 50′, and apply it in the circumference from S. towards E, and the point correſponding to the ſecond bearing will be found, which mark with the figure 2.

4th. The 3d bearing 32° 15′, is to be applied from N. to 3; The 4th bearing 287° 30′, is in the 4th quadrant; therefore take it from 360°, and the remainder 72° 30′, is to be applied from N. towards W. and the point 4 repreſenting the 4th bearing will be known.

In this manner proceed with all the other bearings, and mark the correſponding points in the circumference with the numbers 5, 6, 7, &c. agreeable to the number of the bearing or ſtation.

5th.

5th. Chuſe ſome convenient point on the paper to begin at, as at the place markt ⊙ 1. Lay a parallel ruler by c the centre of the circle, and the point in its circumference marked 1, and (by the help of the ruler) draw a parallel line thro' ⊙ 1, the point choſe for the firſt ſtation, in the direction of the (ſuppoſed) radius C 1; and on this line lay the firſt diſtance; that is, take from a convenient ſized ſcale of equal parts the extent of 1080, and transfer this extent from ⊙ 1 to ⊙ 2; and this line will repreſent the firſt diſtance meaſured, laid down according to its true poſition in reſpect to the circle firſt deſcribed.

6th. Lay the ruler by the centre C, and the point in the circumference noted by the figure 2, and parallel to this poſition of the ruler, draw thro' the point ⊙ 2 a line ⊙ 2 ⊙ 3, in the direction of the (ſuppoſed) radius C 2, and on this line lay from ⊙ 2 to ⊙ 3 the extent 580 taken from the ſame ſcale of equal parts the 1080 was taken from, and this line ſhall repreſent the ſecond meaſured diſtance laid down in its true poſition relative to the firſt diſtance.

Proceed in this manner from ſtation to ſtation until the line ⊙ 7 ⊙ 10 is drawn.

7th. Take from the ſcale of equal parts 314; and apply this extent in the line ⊙ 7 ⊙ 10 from ⊙ 7 to ⊙ 8, and the relative point, where the eighth ſtation was taken, will be repreſented by the point ⊙ 8; then by the parallel ruler draw the line ⊙ 8 ⊙ 5, in the direction of, and parallel to, the (ſuppoſed) radius C 8; and if the preceding work is accurately performed, this line will not only paſs thro' the point ⊙ 5, but the length of the line ⊙ 8 ⊙ 5 will be equal to 510, as the ſtation line was meaſured in the field.

8th. Now as the 9th ſtation falls on the ſame point as the 5th ſtation did, draw the line ⊙ 9 ⊙ 2, and this line will not only be parallel to the (ſuppoſed) radius C 9, but will alſo meaſure on the ſcale of equal parts

parts 390, the length meaſured in the field from the 9th ſtation.

9th. The 10th ſtation is taken from the end of the line 700 meaſured from the 7th ſtation; therefore drawing from ⊙ 10 a line parallel to the (ſuppoſed) radius C 10, this line will concur with the firſt meaſured line at the diſtance of 668 from the point ⊙ 10.

10th Returning to ⊙ 10 again, the ſame point is taken for the 11th ſtation, and the line ⊙ 11 ⊙ 12 is to be drawn parallel to the (ſuppoſed) radius C 11, and to be made of the length of 800 from the ſcale of equal parts; and this will give the point ⊙ 12 for the 12th ſtation: Then drawing the line ⊙ 12 ⊙ 1, if the operation is every where truly done, this line will not only be parallel to the (ſuppoſed) radius C 12, but will alſo meaſure on the ſcale of equal parts 784, the ſame as was meaſured in the field in proceeding from ⊙ 12 to ⊙ 1.

By ſuch methods as theſe, the ſurveyor obtains a cheque on his work, and can make his *ſurvey cloſe* (as 'tis called) as he proceeds.

The drawing of the viſual lines of a ſurvey is, tho' an eſſential part, but a ſmall ſtep towards the making a plan; for the remaining work the reader is refer'd to the treatiſes already extant on that ſubject.

What has been ſaid about the delineating of the viſual lines of a ſurvey, may be applied to navigation in the conſtruction of a figure to repreſent the various courſes and diſtances a ſhip has ſailed in a given time, called traverſe ſailing; for the courſes are the bearings from the Meridian, and the diſtances ſailed are of the ſame kind as the diſtance between ſtation and ſtation in a ſurvey.

Sect.

SECT. XV.

Some Uſes of the Logarithmic Scale of Numbers.

BEFORE any operations can be performed by this ſcale, the notation, or the eſtimating of the values of the ſeveral diviſions, muſt be well known.

Therefore, the Sector being quite opened,

If the 1 at the beginning of the ſcale, or of the 1ſt interval, be taken for	1 10 100 &c. $\frac{1}{10}$ $\frac{1}{100}$ &c.	Then the 1 in the middle, or at the end of the 1ſt interval and the beginning of the ſecond, will expreſs	10 100 1000 &c. 1 $\frac{1}{10}$ &c.	And the 10 at the end of the 2d interval, or end of the ſcale, will repreſent	100 1000 10000 &c. 10 1 &c.

And the primary and intermediate divisions in each interval, must be estimated according to the values set on their extremities, *viz*, at the beginning, middle and end of the scale.

In arithmetical multiplication, or division; the parts may be considered as proportional terms; for in simple multiplication; as unity or 1, is to one factor; so is the other factor, to the product: And in division; as the divisor, is to unity; (or to the dividend,) so is the dividend, (or unity,) to the quotient.

Now as the common logarithms of numbers, express how far the ratios of their corresponding numbers are distant from unity; it follows, that of those numbers which are proportional, that is, have equal ratios; their corresponding logarithms will have equal intervals, or distances: and hence arises the rule for working proportionals on the logarithmic scale.

Rule. Set one foot of the compasses on the point or division representing the first term, and extend the other foot to the point representing the second term: Keep the compasses thus opened; set one foot on the point expressing the third term, and the other foot will fall on the fourth term, or number sought.

Exam. I. *What is the product of* 3 *by* 4?

Solution. Set one foot on the 1 at the beginning, and extend the other to 3, in the first interval; with this opening, set one foot on 4, in the first interval, and the other foot will reach to 12, found in the second interval.

Observe. In this Exam. the 1, 3, and 4, are valued as units in the first interval; and the one in the middle is 10; the distance between this 1 or 10, and the 2 or 20, in the second interval, is divided into 10 principal parts, express'd by the longer strokes; every one in this Exam. is taken as an unit; now as the point of the compasses falls on the second of these principal

principal parts, that is on 2 units beyond 10; therefore this point is to be esteemed in this Exam. as 12.

EXAM. II. *What is the product of* 40 *by* 3?

SOLUTION. In the first interval, take the distance between 1 and 3; and this distance will reach from (4 or) 40 in the first interval to (12 or) 120 in the second interval.

Observe. The 1 and 3 in the first interval, are taken as units: but as the values given to the divisions in either interval, may as well be call'd 40, as 4; and being taken as 40, the 1 at the beginning of the second interval will be 100; and the 2 in the second interval will be 200: consequently the principal divisions between this 1 and 2 will each express 10; and so the second of them will be 20, which with the 100, express'd by the 1, makes 120.

EXAM. III. *What is the product of* 35 *by* 24?

SOLUTION. The distance from 1 in the first interval, to 24 in the second, will reach from 35 in the first interval, to 840 in the second.

Observe. In the first application of the compasses, the primary divisions in the first interval are taken as units, and those in the second interval, as tens: But in the second application, the primary divisions in the first interval are reckon'd as tens; and those in the second, as hundreds.

As the extent out of one interval into the other, may sometimes be inconvenient, it will be proper to see in such cases, how the example may be solved in one interval. Thus,

IN either interval, take the extent from 1 to $2\frac{4}{10}$ (i. e. 24) and this extent, (in either interval) will reach from $3\frac{5}{10}$ (i. e. 35) to $8\frac{4,0}{10,0}$; (i. e. 840.)

In this operation; the ſecond term is reckoned a tenth higher than the firſt term; therefore, as it falls in the ſame interval, the fourth term muſt be a tenth higher than the third term.

Exam. IV. *What is the product of* 375 *by* 60?

Solution. The extent from 1 to 6, (or 60) in the firſt interval will reach from $3\frac{7\frac{1}{2}}{10}$ ($=3\frac{75}{100}$ or 375) in the firſt interval, to $2\frac{25}{100}$ in the ſecond interval; which diviſion muſt be reckoned 22500: For had the point fell in the firſt interval, it would have been one place more than the 375, becauſe 60 is one place more than 1; but as it falls in the ſecond interval, every of whoſe diviſions is one place higher than thoſe in the firſt interval, therefore, it muſt have two places more than 375, which is taken in the firſt interval.

If the operations in theſe examples be well conſidered, it will not be difficult to apply others to the ſcale, and readily to aſſign the value of the reſult.

Exam. V. *What will be the quotient of* 36 *divided by* 4?

Solution. The extent from 4 to 1, in the firſt interval, will reach from 36 in the ſecond interval to nine in the firſt.

It is to be obſerved, that when the ſecond term is greater than the firſt term; the extents are reckoned from the left hand towards the right: and when the ſecond term is leſs than the firſt, the extents are taken from the right hand towards the left: that is, the extents are always counted the ſame way towards which the terms proceed.

Exam.

EXAM. VI. *If* 144 *be divided by* 9; *what will be the quotient?*

SOLUTION. The extent from 9 to 1, will reach from 144 to 16 - 144 to 16

EXAM. VII. *If* 1728 *be divided by* 12; *what will be the quotient?*

SOLUTION. The extent from 12 to 1, will reach from 1728 to 144.

EXAM. VIII. *To the numbers* 3, 8, 15; *find a fourth proportional.*

SOLUTION. The extent from 3 to 8; will reach from 15 to 40.

EXAM. IX. *To the numbers* 5, 12, 38; *find a 4th proportional.*

SOLUTION. The extent from 5 to 12, will reach from 38 to 91 $\frac{1}{5}$.

EXAM. X. *To the numbers* 18, 4, 364; *find a 4th proportional.*

SOLUTION. The extent from 18 to 4; will reach from 364 to 80 $\frac{8}{9}$.

EXAM. XI. *To two Numbers* 1 *and* 2; *to find a series of continued proportionals.*

SOLUTION. The extent from 1 to 2, will reach from 2 to 4; from 4 to 8 in the first interval; from 8 to 16 in the second interval; from 16 to 32; from 32 to 64; *&c.* Also the same extent will reach from 1 $\frac{1}{2}$ to 3; from 3 to 6; from 6 to 12; from 12 to 24; from 24 to 48; *&c.* And the same extent will reach from 2 $\frac{1}{2}$ to 5; from 5 to 10; from 10 to 20; from 20 to 40; *&c.* And in a like manner proceed, if any other ratio was given besides that of 1 to 2.

This Example is of uſe, to find if the diviſions of the line of numbers, are accurately laid down on the ſcale.

There are many other uſes to which this ſcale of log. numbers are applicable, and on which ſeveral large treatiſes have been wrote; but the deſign here, is not to enter into all the uſes of the ſcales on the ſector, only to give a few examples thereof: but after all that has been ſaid, when examples are to be wrought whoſe reſult exceeds three places, 'tis beſt to do it by the pen, for on inſtruments, altho' they be very large ones, the loweſt places of the anſwers, at beſt, are but gueſs'd at.

Sect. XVI.

Some uſes of the Scales of Log. Sines and Log. Tangents.

THESE ſcales are chiefly uſed in the ſolution of the caſes of plain and ſpherical trigonometry, which will be fully exemplified hereafter: But in this place, it will be proper to ſhew, how proportional terms are applied to the ſcales.

In plane trigonometrical proportions, there are always four terms under conſideration; ſuppoſe two ſides and two angles, whereof, only three of the terms are given, and the fourth is required: Now the ſides in plane trigonometry, are always applied to the ſcale of log. numbers; and the angles are either applied to the log. ſines, or to the log. tangents; according as the ſines or tangents are concerned in the proportion. Therefore, when among the three things given, if two of them be ſides, and the other an angle; or if two terms be angles, and the other a ſide.

Rule.

Rule. On the ſcale of log. numbers, take the extent between the diviſions expreſſing the ſides; and this extent applied from the diviſion expreſſing the angle given, will reach to the diviſion ſhewing the angle required.

Or, the extent of the angles taken, will reach from the ſide given to the ſide required, on the line of numbers.

So in ſpherical trigonometry, where ſome of the caſes are worked wholly on the ſines, others partly on ſines, and partly on tangents; the extent taken with the compaſſes, between the firſt and ſecond terms, when thoſe terms are of the ſame kind, will reach from the third term to the fourth.

Or, the extent from the firſt term to the third, when they are of the ſame kind, will reach from the ſecond term to the fourth.

Sect. XVII.

Some uſes of the double Scales of Sines, Tangents, and Secants.

PROBLEM XX.

Given the radius of a circle (ſuppoſe equal to 2 inches) required the ſine, and tangent of 28° 30′ to that radius.

Solution. Open the ſector ſo that the tranſverſe diſtance of 90 and 90, on the ſines; or of 45 and 45 on the tangents; may be equal to the given radius; *viz.* two inches: Then will the tranſverſe diſtance of 28° 30′, taken from the ſines, be the length of that ſine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of 28° 30′ *was required?*

MAKE the given radius two inches, a transverse distance to 0 and 0, at the beginning, of the line of secants; and then take the transverse distance of the degrees wanted, *viz.* 28° 30′.

A Tangent greater than 45 *degrees (suppose* 60 *degrees) is found thus.*

MAKE the given radius, suppose 2 inches, a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required degrees 60° 00′ may be taken from this scale.

The scales of upper tangents and secants do not run quite to 76 degrees; and as the tangent and secant may be sometimes wanted to a greater number of degrees than can be introduced on the sector, they may be readily found by the help of the annexed table of the natural tangents and secants of the degrees above 75; the radius of the circle being unity.

Degrees.	Nat. Tangent.	Nat. Secant.
76	4,011	4,133
77	4,331	4,445
78	4,701	4,810
79	5,144	5,241
80	5,671	5,759
81	6,314	6,392
82	7,115	7,185
83	8,144	8,205
84	9,514	9,567
85	11,430	11,474
86	14,301	14,335
87	19,081	19,107
88	28,636	28,654
89	57,290	57,300

Measure

Meaſure the radius of the circle uſed, upon any ſcale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or ſecant ſought, to be taken from the ſame ſcale of equal parts.

EXAM. *Required the length of the tangent and ſecant of* 80 *degrees to a circle whoſe radius, meaſured on a ſcale of* 25 *parts to an inch, is* $47\frac{1}{2}$ *of thoſe parts?*

	tangent.	ſecant.
Againſt 80 degrees ſtands	5,671	5,759
The radius is	47,5	47,5
	28355	28795
	39697	40313
	22684	23036
	269,3725	273,5525

So the length of the tangent on the twenty-fifth ſcale will be $269\frac{1}{3}$ nearly. And that of the ſecant about $273\frac{1}{2}$.

OR thus. The tangent of any number of degrees may be taken from the ſector at once; if the radius of the circle can be made a tranſverſe diſtance to the complement of thoſe degrees on the lower tangent.

EXAM. *To find the tangent of* 78 *degrees to a radius of* 2 *inches.*

MAKE two inches a tranſverſe diſtance to 12 degrees on the lower tangents; then the tranſverſe diſtance of 45 degrees will be the tangent of 78 degrees.

IN like manner the ſecant of any number of degrees may be taken from the ſines, if the radius of the circle can be made a tranſverſe diſtance to the coſine of thoſe degrees. Thus making two inches a tranſverſe diſtance to the ſine of 12 degrees; then the tranſverſe diſtance of 90 and 90 will be the ſecant of 78 degrees.

FROM hence it will be eaſy to find the degrees anſwering to a given line, expreſſing the length of a

tangent or secant, which is too long to be measured on those scales, when the sector is set to the given radius.

Thus. For a tangent, make the given line a transverse distance to 45 and 45 on the lower tangents; then take the given radius and apply it to the lower tangents; and the degrees where it becomes a transverse distance is the cotangent of the degrees answering to the given line.

And for a secant. Make the given line a transverse distance to 90 and 90 on the sines. Then the degrees answering to the given radius applied as a transverse distance on the sines, will be the co-sine of the degrees answering to the given secant line.

PROBLEM XXI.

Given the length of the sine, tangent, or secant, of any degrees; to find the length of the radius to that sine, tangent, or secant.

Make the given length, a transverse distance to its given degrees on its respective scale: Then,

In the sines. The transverse distance of 90 and 90 will be the radius sought.

In the lower tangents. The transverse distance of 45 and 45 near the end, of the sector will be the radius sought.

In the upper tangents. The transverse distance of 45 and 45 taken towards the centre of the sector on the line of upper tangents, will be the centre sought.

In the secants. The transverse distance of 0 and 0, or the beginning of the secants, near the centre of the sector, will be the radius sought.

PROB-

PROBLEM XXII.

Given the radius and any line representing a sine, tangent or secant; to find the degrees corresponding to that line.

Solution. Set the sector to the given radius, according as a sine, or tangent, or secant is concerned.

Take the given line between the compasses; apply the two feet transversly to the scale concerned, and slide the feet along till they both rest on like divisions on both legs; then will those divisions shew the degrees and parts corresponding to the given line.

PROBLEM XXIII.

To find the length of a versed sine to a given number of degrees, and a given radius.

Make the transverse distance of 90 and 90 on the sines, equal to the given radius.

Take the transverse distance of the sine complement of the given degrees.

If the given degrees are less than 90, the difference between the sine complement and the radius, gives the versed sine.

If the given degrees are more than 90, the sum of the sine complement and the radius, gives the versed sine.

PROBLEM XXIV.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, tangents, may make, each, a right angle.

On the lines, make the lateral distance 10, a distance between 8 on one leg, and 6 on the other leg.

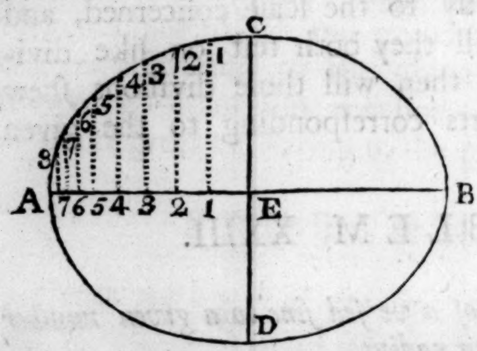
On the sines. make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from

from 30 to 60 ; or from the ſine of any degrees, to their complement.

Or on the ſines, make the lateral diſtance of 45 a tranſverſe diſtance between 30 and 30.

PROBLEM XXV.

To deſcribe an Ellipſis, having given AB *equal to the longeſt diameter* ; *and* CD *equal to the ſhorteſt diameter.*



SOLUTION. 1ſt. Set the two diameters AB, CD, at right angles to each other in their middles at E.

2d. MAKE AE a tranſverſe diameter to 90 and 90 on the ſines ; and take the tranſverſe diſtances of 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, ſucceſſively, and apply thoſe diſtances to AE from E towards A, as at the points 1, 2, 3, 4, 5, 6, 7, 8 ; and thro' thoſe points draw lines parallel to EC.

3d. MAKE EC a tranſverſe diſtance to 90 and 90 on the ſines ; take the tranſverſe diſtances of 80°, 70°, 60°, 50°, 40°, 30°, 20°, 10°, ſucceſſively, and apply thoſe diſtances to the parallel lines from 1 to 1, 2 to 2, 3 to 3, 4 to 4, 5 to 5, 6 to 6, 7 to 7, 8 to 8, and ſo many points will be obtained thro' which the curve of the ellipſis is to paſs,

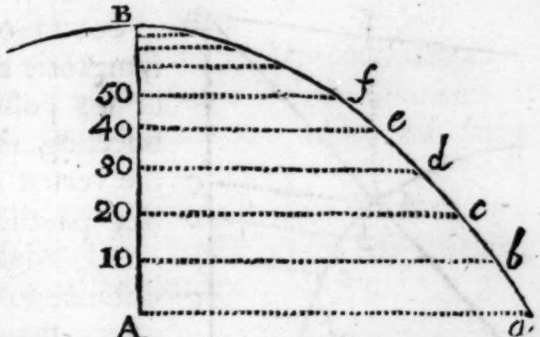
The ſame work being done in all the four quadrants, the elliptical curve may be compleated.

This Problem is of conſiderable uſe in the conſtruction of ſolar Eclipſes ; but inſtead of uſing the ſines to every ten degrees, the ſines belonging to the degrees and minutes correſponding to the hours, and quarter hours are to be uſed.

To

PROBLEM XXVI.

To describe a Parabola whose parameter shall be equal to a given line.



SOLUTION 1st. Draw a line to represent the axis, in which make AB equal to half the given parameter; divide AB like a line of sines to every ten degrees, as at the points 10, 20, 30, 40, 50, &c. and thro' these points draw lines at right angles to the axis AB.

2d. MAKE the lines A*a*, 10*b*, 20*c*, 30*d*, 40*e*, &c. respectively equal to the chords of 90° 80° 70°, 60°, 50°, &c. to the radius AB, and the points *a*, *b*, *c*, *d*, *e*, &c. will be in the curve of a parabola.

Therefore a smooth curve line drawn thro' those points and the vertex B, will represent the parabolic curve required.

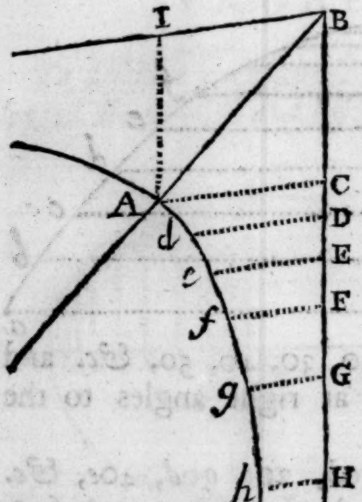
The like work may be done on both sides of the axis when the whole curve is wanted.

As the chords on the sector run no farther than 60, those of 70, 80 and 90 may be found by taking the transverse distance of the sines of 35°, 40°, 45° to the radius AB, and applying those distances twice along the lines 20*c*, 10*b*, &c.

PRO-

PROBLEM XXVII.

To describe an Hyperbola, the vertex A *and assymptotes* BH, BI, *being given.*



SOLUTION 1st. The assymptotes BH, BI, being drawn in any position, the line BA, bisecting the angle IBH, and the vertex A taken, draw AI, AC, parallel to BH, BI.

2d. Make AC a transverse distance to 45 and 45 on the upper tangents, and apply to the assymptotes from B, so many of the upper tangents taken transversly as may be thought convenient, as BD 50°, BE 55°, BF 60°, BG 65°, BH 70°, &c. and draw D*d*, E*e*, &c. parallel to AC.

3d. Make AC a transverse distance to 45 and 45 on the lower tangents, take the transverse distance of the co-tangents before used, and lay them on those parallel lines; thus make D*d*=40°, E*e*=35°, Ff=30°, G*g*=25°, H*h*=20°, &c. and thro' the points A, *d*, *e*, *f*, *g*, *h*, &c. If a curve line be drawn it will be the hyperbola required.

There are many other methods of constructing the curves in the three last problems, and a multitude of entertaining and useful properties which subsist among the lines drawn within and about these curves, which the inquisitive reader will find in the treatises on conic sections.

PRO-

PROBLEM XXVIII.

To find the distance of places on the terrestrial globe by having given their latitudes and longitudes.

This problem consists of six cases.

CASE I. If both the places are under the equator.

Then the difference of longitude is their distance.

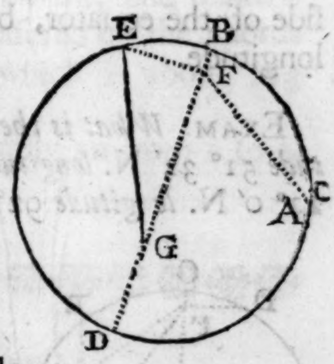
CASE II. When both places are under the same meridian.

Then the difference of latitude is their distance.

CASE III. When only one of the places has latitude, but both have different longitudes.

EXAM. *Island of* Bermudas, *lat.* 32° 25′ N. *longit.* 68° 38′ W. *Island of* St. Thomas, *lat* 0 0, *longit.* 1° 0 E.
Required their distance.

SOLUTION 1st. With the chord of 60° describe a circle representing the equator, wherein take a point C to represent the beginning of longitude.



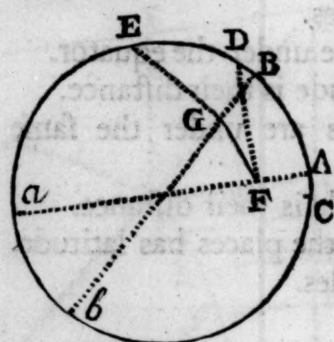
2d. From C apply the chord of *Bermudas* longitude 68° 38′ to B, and that of *St. Thomas*'s longitude to A, the arc AB, being the difference of longitude.

3d. From B, the place having latitude, draw the diameter BD, apply the chord of the latitude 32° 25, from B to E, and draw EF at right angles to BD.

4th. Draw FC, make FG, equal to FC, and draw EG; then EG measured on the chords will give the distance sought, about 73 degrees.

CASE IV. When the given places are in the same parallel of latitude.

EXAM. *Required the distance between the* Lizard *and* Pengwin *Island, both in latitude* 49°, 56′ N. *the longitude of the* Lizard *being* 5° 14′ W. *and that of* Pengwin *Island* 50° 32′ W.

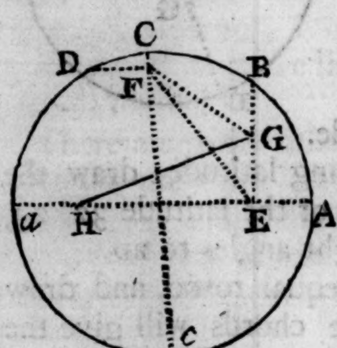


SOLUTION 1st. From C, the commencement of the longitude, apply the chord of the *Lizard*'s longitude to A, and of *Penguin*'s longitude to B, and draw the diameters A*a*, B*b*.

Apply the chord of the common latitude 49° 56′ from A to D, and from B to E; draw DF and EG at right angles to A*a*, B*b*, and join GF; then GF measured on the chords will give the distance sought, about 29 degrees.

CASE V. When the given places are on the same side of the equator, but differ both in latitude and longitude.

EXAM. *What is the distance between* London *in latitude* 51° 32′ N. *longitude,* 0° 0′ *and* Bengal *in latitude* 22° 0′ N. *longitude* 92° 45′ E.



SOLUTION. From A, *London*'s longitude, apply *Bengal*'s longitude 92° 45′ to C, taken from the chords; also apply the chord of *London*'s latitude from A to B, and of *Bengal*'s latitude from C to D.

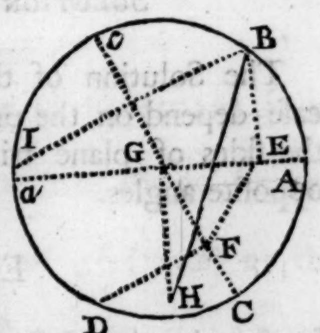
2d. Draw the diameters A*a*, C*c*, and BE, DF, at right angles to A*a*, C*c*, and join FE.

3d. Make BG equal to DF, and EH equal to EF, join GH; Thus GH measured on the chords will give the distance required, which is about 72 degrees.

CASE

Case VI. When the places are on contrary sides of the equator, and differ both in latitude and longitude.

Exam. *What is the distance between* London *in latitude* 51° 32′ N. *longitude* 0° 0′ *and* Cape-Horn *in latitude* 55° 42′ S. *longitude* 66° 00′ W.



Solution 1st. From A, *London*'s longitude, apply the chord of *Cape-Horn*'s longitude to C, draw the diameters A*a*, C*c*; also apply the chords of *London*'s latitude from A to B, and of *Horn*'s latitude from C to D.

2d. Draw BE and DF at right angles to A*a*, C*c*; join EF and make EG equal to EF.

3d. At right angles to A*a*, draw GH, and make it equal to DF; join BH, which measured on the chords will give the distance required, which is about 123 degrees.

To measure BH *on the chords*; apply BH from B to I, and measure the arc B*c* I.

Sect. XVII.

The Use of some of the single and double Scales, applied in the Solution of the Cases of plain Trigonometry.

PROBLEM XXIX.

In any right lin'd plane triangle, any three of the six terms, viz. *sides and angles, (provided one of them be a side) being given, to find the other three.*

This

This problem consists of three cases.

Case I. when among the things given, there be a side and its opposite angle.

Case II. When there is given two sides and the included angle.

Case III. When the three sides are given.

Solution *of* CASE I.

The Solution of the examples falling under this case depend on the proportionality there is between the sides of plane triangles, and the sines of their opposite angles.

Example I.

In the triangle ABC: Given $\left.\begin{array}{l}AB=56\\AC=64\end{array}\right\}$ equal parts.

$\angle B=46^\circ\ 30'$

Required $\angle C$, $\angle A$, & BC.

The proportions are as follow,

As side AC : side AB :: sine $\angle$ B : sine $\angle$ C.

Then the sum of the angles B and C being taken from 180°, will leave the angle A.

And as sine $\angle$B : sine $\angle$A :: side AC : side CB.

First by the logarithm scales.

To find the angle C.

The extent from 64 (=AC) to 56 (=AB) on the scales of logarithm numbers, will reach from 46° 30′ (=$\angle$B) to 39° 24′, (=$\angle$C.) on the scale of logarithm sines.

And the sum of 46° 30′ and 39° 24′ is 85° 54′

Then 85° 54′ taken from 180°, leaves 94° 6′ for the angle A.

To

To find the side BC.

The extent from 46° 30′ (=∠B) to 85° 54′ the supplement of 94° 6′ (=∠A) on the scale of log. sines, will reach from 64 (=AC), to 88 (=BC), on the scale of logarithm numbers.

Secondly by the double Scales.

To find the Angle C.

1. Take the lateral distance of 64 (=AC) from the lines.

2. Make this a transverse distance of 46° 30′ (=∠B) on the sines.

3. Take the lateral distance of 56 (=AB) on the lines.

4. Find the degrees to which this extent is a transverse distance on the sines, *viz.* 39° 24′; and this is the angle sought.

To find the side BC.

1. Take the lateral distance of 64 (=AC) from the lines.

2. Make this a transverse distance of 46° 30′ (=∠B) on the sines.

3. Take the transverse distance of 85° 54′ (the supplement of 94° 6′ = ∠ A) on the sines.

4. Find the lateral distance this extent is equal to, on the lines; and this distance, *viz.* 88, will be the side required.

Ex. II. In the triangle ABC Pl. VI. Fig. 27.

Given BC = 74
∠B = 104° 0′
∠C = 28 0
Required AB & AC.

Now the sum of 104° 0′ and 28° 0′ is 132° 0′.

And 132° 0′ taken from 180, leaves 48° 0′ for the angle A.

The proportions are,
As ſine ∠A : ſine ∠C : : ſide BC : ſide AB.
And as ſine ∠A : ſine ∠B : : ſide BC : ſide AC.

First, by the Logarithm Scales.

To find AB.

THE extent from 48° 0′ (= ∠A) to 28° 0′ (=∠C) on the ſcale of logarithm ſines, will reach from 74 (=BC) to 46, 75, (=AB,) on the ſcale of logarithm numbers.

To find AC.

THE extent from 48° 0′ to 76° 0′ (= ſupplement of 104° 0′) on the ſcale of log. ſines, will reach from 74 to 96, 0 (=AC) on the ſcale of logarithm numbers.

Secondly by the double Scales.

To find AB.

1. TAKE the lateral diſtance 74 (= BC) on the lines.
2. MAKE this extent a tranſverſe diſtance to 48° 0′ (= ∠A) on the ſines.
3. TAKE the tranſverſe diſtance of 28° 0′ (=∠C) on the ſines.
4. To this extent find the lateral diſtance on the lines, *viz.* 46,75 and this will be the length of AB.

To find AC.

1. TAKE the lateral diſtance 74 (= BC) on the lines.
2. MAKE this extent a tranſverſe diſtance to 48° 0′ (= ∠A) on the ſines.

3. TAKE the transverse distance to $76^\circ\ 0'$ the supplement of $104^\circ\ 0'$ ($= \angle B$) on the sines.

4. To this extent, find the lateral distance on the lines, *viz.* 96, 6, and this will be the length of AC.

SOLUTION *of* CASE II.

THE solution of this case depends on a well known theorem, *viz.*

As the sum of the given sides
Is to the difference of those sides,
So is the tangent of the half sum of the unknown angles
To the tangent of the half difference of those angles.

And the angles are readily found by their half sum and half difference being known.

Ex. III. In the triangle ABC, Pl. VI. Fig. 28.
Given $BC = 74$
$BA = 52$
$\angle B = 68^\circ\ 0'$
Required $\angle A$; $\angle C$; & AC.

Preparation.

TAKE the given angle $68^\circ\ 0'$ from 180°, and half the remainder, *viz.* $56^\circ\ 0'$ is the half sum of the unknown angles which call z; and let x stand for the half difference of those angles.

ALSO find the given sum of the sides, *viz.* $BC+BA = 126$.

AND take the difference of those sides, *viz.* $BC-BA=22$.

Then the proportions are

As $BC+BA : BC-BA :: tan.\ z : tan.\ x$.

THEN the sum of z and x gives the greater angle A.

THE difference of z and x gives the leſſer angle C,
AND as ſine ∠C : ſine ∠B : : ſine BA : ſide AC.

First by the Logarithm Scales.

To find the tangent of N.

TAKE the extent from 126 (= ſum of the given ſides) to 22 (= diff. of thoſe ſides) on the ſcale of logarithm numbers; lay this extent from 45° 0′ to the left on the logarithm tangents; ſtay the loweſt point, and bring that which reſted on 45 degrees, to 56° 0′; remove the compaſs, and this extent laid from 45° 0′ towards the left, gives 14 . 31′ equal N.

THEN the ſum of 56° 0′ and 14° 31′ or 70° 31′ is the angle A.

AND 14° 31′ taken from 56° 0′ leaves 41° 29′ for he angle C.

To find AC.

THE extent from 41° 29′ (= ∠C) to 68° 0′ (= ∠B) on the logarithm ſines, will reach from 52 (= BA) to 72, 75 (= AC) on the ſcale of logarithm numbers.

IN finding the tangent of (N, or) the half difference of the unknown angles, there were two applications of the compaſſes to the ſcale of tangents: Now this happens becauſe the upper tangents which ſhould have been continued beyond 45°, or to the right hand, are laid down backwards, or to the left hand, among the lower tangents (the logarithmic tangents aſcending and deſcending by like ſpaces at equal diſtances on both ſides of 45°), and thereby the length of the ſcale is kept within half the length neceſſary to lay down all the tangents in order, from the left towards the right. But ſuppoſing they were ſo laid down, then the point of 56° 0′ will reach as far to the right of 45° as it does now to the left, and the

the extent on the numbers from 126 to 22 would reach from the point 56° taken on the right of 45°, to 14° 31′ at one application; the said extent being applied from 45° downwards, will reach as far beyond 14° 31′, as is the distance from 45° to 56°; therefore the legs of the compasses being brought as much closer as is that interval, will reach from 45° to the degrees wanted.

INDEED when the half sum is less than 45°, then the extent from the sum of the sides to their difference, will reach from the tangent of the half sum, downward, to the tangent of the half difference, at once.

AND when the half sum of the unknown angles, and their half difference, are both greater than 45°, then the extent from the sum of the sides to their difference, will reach from the tangent of the half sum of the angles, upwards (or to the right) to the tangent of the half difference of those angles, at once.

Secondly by the double Scales.

Because 126 the sum of the sides will be longer than the scales of lines, therefore take 63, the half of 126, and 11, the half of 22, the difference of the sides; for the ratio of 63 to 11, is the same as that of 126 to 22. Then

1. TAKE the lateral distance 63 on the scales of lines.

2. MAKE this extent a transverse distance to 56 degrees, on the upper tangents.

3. TAKE the transverse distance of 45° on the upper tangents, and make this extent a transverse distance to 45° on the other tangents.

4. TAKE the lateral distance 11, on the lines.

5. To this extent, find the transverse distance on the tangents, and this will be 14° 31′ = N.

AND this is the manner of operation, when M is greater than 45 degrees, and N is less.

But when M & N are each $\left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right\}$ than 45 degrees.

Then the third article of the foregoing operation is omitted.

Now having found the angles A and C, the side AC may be found as in the first or second examples.

But in this case, the third side AC may be found without knowing the angles. Thus,

1. Take the lateral distance of (34 deg.) the half of (68,) the given angle, from the sines.
2. Make this extent a transverse distance, to 30 on the sines.
3. With the sector thus opened, take the distance from 74 on one leg, to 52 on the other leg, each reckon'd on the lines.
4. The lateral distance, on the lines, of this extent, gives the side AC = 72, 75.

From the two first articles of this operation, is learn'd how to set the double scales to any given angle.

When the included angle B is 90 degrees, the angles A and C are more readily found, as in the following example, whose solution depends on this principle. That one of the given sides has the same proportion to radius, as the other given side has to the tangent of its opposite angle.

Ex. IV. In the triangle ABC: Fig. 29.

Given AB = 45
BC = 65
∠B = 90
Required ∠A ; ∠C ; & AC.

The proportions are,

For the Angle A.

As side AB : side BC : : radius : *tan.* ∠ A.

And

And the $\angle A$ taken from 90° leaves the $\angle C$, then AC may be found as directed in the last example.

First by the logarithmic Scales.

The extent from 45 (= AB) to 65 (= BC) on the numbers, will reach from 45 degrees to $55^\circ\ 18'$ ($= \angle A$) on the tangents.

Here the angle A is taken equal to $55^\circ\ 18'$, because the second term BC is greater than the first term AB : But if the terms were changed, and it was made BC to AB, then the degrees found would be $34^\circ\ 42'$ $= \angle C$.

Secondly by the double Scales.

1. Take the lateral distance of the first term, from the lines.

2. Make this a transverse distance to 45 deg. on the tangents.

3. Take the lateral distance of the second term, from the lines.

4. The transverse distance of this extent, found on the tangents, gives the degrees in the angle sought.

If the first term is greater than the second, then the lateral distance of the first term, must be set to 45 degrees on the lower tangents, and the lateral distance of the second term, must be reckon'd on the same tangents.

But if the first term is less than the second, then the lateral extent of the first term must be set to 45° on the upper tangents, and the lateral extent of the second term must be reckon'd on the same tangents.

Solution *of* CASE III. Fig. 30.

In the triangle ABC :

Given BC = 926.
BA = 558.
AC = 702.
Requir'd $\angle B$, $\angle C$. $\angle A$.

 There

There are usually given for the solution of this case by the logarithmic scales two methods; the one best when all the angles are to be found, the other best when one angle only is wanted; both methods will be here delivered.

First. *When all the angles are wanted.*

Suppose a perpendicular AD (Pl. VI. Fig. 30.) drawn to the greatest side BC, from the angle A opposite thereto; then AD divides the triangle ABC into two right angled triangles BDA, CDA; in which if CD and DB were known, the angles would be found, as in the solution of Case I.

Take the sum of the sides AC and AB, which is 1260.

Also their difference, which is 144.

Then on the scale of numbers, the extent from 926 (= BC) to 1260, will reach from 144 to 196.

And the half sum of 926 and 196, is 561 = DC.

And the half difference of 926 and 196 is 365 = DB.

The extent from 558 (= BA) to 365 (= BD) on the numbers, will reach on the log. sines from 90° (= ∠BDA) to 40° 52′ (= ∠BAD.)

Then 40° 52′ taken from 90°, leaves 49° 8′ for ∠B.

And the extent from 702 (= CA) to 561 (= CD) on the numbers, will reach from 90° (= ∠CDA) to 53° 04′ (= ∠CAD) on the scale of log. sines.

Then 53° 4′ taken from 90°, leaves 36° 56′ for the ∠C.

Also the sum of 40° 52′ and 53° 4′ gives 93° 56′ for the ∠CAB.

Secondly,

SECONDLY, *To find either angle; suppose* B.

Preparation.

TAKE the difference between BC and BA, the sides including the angles sought, and call it D = 368.

FIND the half sum of AC and D, call it Z = 535

AND the half diff. of AC and D, call it X = 167

THEN as I : $\sqrt{\frac{Z \times X}{AB \times BC}}$: : radius : sine $\frac{1}{2}$ ∠B.

1. THE extent on the log. numbers from I to 535 (= Z), will reach from 167 (= X) to a 4th point; mark it and call it G.

2. THE extent from I to 558 (= AB), will reach from 926 (= BC) to a 4th point; mark it and call it H.

3. THE extent from the point H to the point G, will reach from I, downward to a 4th point, mark it and call it K.

4. THE extent from K, to the middle point between it and the I next above K, taken on the log. numbers, will reach on the log. sines from 90° to 24° 34′, which doubled gives 49° 8′ for the angle B.

BUT the scale of log. versed sines being used, the work will be considerably shortened. Thus,

1. ON the log. numbers take the extent from 535 (=Z) to 926 (= BC), this will reach from 558 (=BA) to a 4th point, where let the foot of the compasses rest.

2. THEN the extent from that 4th point to 167 (=X), will reach on the line of versed sines from 0 degrees (at the end) to 130° 52′, which taken from 180° leaves 49° 8′ for the angle B.

By

By the double, or sectoral, Scales.

To find the angle B.

1. TAKE the lateral distance 702 (= AC, the side opposite to the angle B) from the lines.

2. OPEN the legs of the sector until this extent will reach from 926 (= CB) on one scale of lines, to 558 (= AB) on the other scale of lines.

3. THE sector being thus opened, take the transverse distance between 30° and 30° on the sines, this distance measured laterally on the sines, one foot being on the centre, will give 24° 34′ for half the angle B.

THE other angles may be found as ∠B was, or according to the directions in some of the preceding cases.

ALTHOUGH in these examples, oblique triangles were taken as being the most general; yet it may be readily seen, that those concerning right-angled triangles are only particular cases, and may be, for the general, more easily solved.

VARIETY of other examples, shewing the uses of these scales, might be given in various parts of the mathematics, which the reader may of himself supply: However here will be subjoined a few in spherical trigonometry, as they will include some operations not only curious, but perhaps not to be met with elsewhere.

SECT.

S E C T. XVIII.

The Conſtruction of the ſeveral caſes of Spherical Triangles by the Scales on the Sector.

THE caſes of ſpherical triangles are ſix.

CASE I. Given two ſides, and an angle oppoſite to one of them.

CASE II. Given two angles, and a ſide oppoſite to one of them.

CASE III. Given two ſides, and the included angle.

CASE IV. Given two angles, and the included ſide.

CASE V. Given the three ſides.

CASE VI. Given the three angles.

Theſe ſix caſes include all the variety that can ariſe in ſpherical triangles.

IN the following ſolutions, are given three conſtructions to every caſe, whereby each ſide is laid on the plane of projection, or (as it is commonly called, the) primitive circle.

To abbreviate the directions given in the following conſtructions, it is to be underſtood, that the primitive circle is always firſt deſcribed, and two diameters drawn at right angles.

THE ſector is alſo ſuppoſed to be ſet to the radius wanted, on the ſcale uſed ; and the tranverſe diſtance of the degrees propoſed is to be taken from the chords, or ſecants, or tangents, &c. according to the name mentioned in the conſtruction.

So-

SOLUTION *of* CASE I.

EXAM. In the spherical triangle ABD.

Given AB = 29° 50′
DB = 63 59
∠D = 25 55

Required the triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 1. Pl. VII.

1st. Make DB = chord of 63° 59′, and draw the diameter BE.

2d. From D, with the secant of the ∠ D, 25° 55′, cut the diameter ⊙ I in C: on C as a center, with that radius, describe the circumference DA, and the angle BDA will be 25° 55′.

3d. Make B*d* equal to AB, with the chord of 29° 50′.

4th. With the tangent of AB, 29° 30′, from *d*, cut ⊙ B produced in *b*; and from *b*, with that radius, cut DA in A or *a*.

5th, Through B, A, E, describe a circumference, and the triangle BDA will be that required; whose parts DA, ∠B, and ∠A may be thus measured.

To measure DA.

6th. Make ⊙ P equal to the tangent of half the angle BDA, *viz.* 12° 57½′; then a ruler on P and A, gives *e*; and D *e* measured on the chords, gives the degrees in DA, *viz.* 42° 9′.

To measure ∠B.

7th. Draw the diameter FG at right angles to BE, cutting the circumference BAE in *s*; A ruler by B & *s* gives *f*; make *fg* equal to the chord of 90 deg.

a ruler on *g* and B, gives *p* in the diameter FG. Then E *g* on the chords gives the angle B = 36° 9′.

To measure ∠A.

8th. A ruler on A and P, gives *n*; and on A and *p*, gives *m*; and *nm* measured on the chords, gives 52° 9′, for the supplement of the angle DAB, which is 127° 51′.

II. *To put* DA *on the primitive circle*, Fig. 2. I.

1st. With the secant of the angle D, 25°, 55′, from D, cut the diameter in C; and on C, with the same radius, describe the arc DB, and the angle BDA will be 25°, 55′.

2d. Make ☉ P, equal to the tangent of half the angle D; *viz.* 12° 57′ ½.

3d. On the primitive circle, make D *d* equal to the given side DB, with the chord of 63° 59′.

4th. A ruler on B and *d*, gives B; then will BD = 63° 59′.

5th. Draw ☉ B *r*, cutting the primitive circle in *r*.

6th. Make *r x* = the chord of 90°; or twice the chord of 45°.

7th. A ruler on *x* and B, gives *m* on the primitive circle.

8th. Make *m q* = *mp* = chord of 29° 50′.

9th. A right line through *x* & *p*, *x* & *q*, gives *f* & *e* in ☉ *r*.

10th. On *fe* as a diameter, describe a circumference, cutting the primitive circle in A, *a*.

11th. A ruler on A & ☉, gives F.

12th. Through A, B, F, describe a circumference, and the triangle ABD is constructed with DA on the primitive circle as required.

III. *To put* AB *on the primitive circle.* Fig. 3. I.

1st. Make AB = the chord of 29° 50′; and draw the diameter BF.

2d. In

2d. In A b drawn perpendicular to AG, take A b = sine of AB 29° 50′.

3d. Make the angle b A g, = ∠ D 25° 55′; from A draw A e at right angles to A g, and from d, the middle of A b, draw de perpendicular to A b, cutting Ae, in e; from e, with the radius e A, describe a circumference A f b.

4th. From b, with the sine of BD, 63° 59′, cut the circumference Afb in f; and draw A f.

5th. From A, draw AC at right angles to f A, meeting E ⊙ (perpendicular to A ⊙,) continued, in C; and on C, with the radius CA, describe a circumference ADG.

6th. Make Bm= BD, with the chord of 63° 59′; from m, with the tangent of 63° 59′ cut ⊙ B produced, in n; on n, with the same radius, cut ADG in D.

7th. Through B, D, F, describe a circumference, and the triangle ABD will be that which was required.

Computation by the logarithmic scales.

To find the angle A.

The sines of the angles of spheric triangles are as the sines of their opposite sides.

Then the extent of the compasses on the line of sines from 29° 50′ (= AB) to 25° 55′ (= ∠ C); will reach from the sine of 63° 59′ (= CB) to the sine of 52° 9′ (= ∠ A).

But by construction the ∠ A is obtuse; therefore 127° 51′ (the supplement of 52° 9′) is to be taken for the angle A.

To find the angle B.

Say, as radius, to the cosine of CB.
So tang. ∠C, to the cotang of a fourth arc.
And as tang. AB, to the tang. of CB.
So cosine of the 4th arc, to the cosine of a 5th arc.

Then

Then the difference between the 4th and 5th arcs gives ∠ B.

The extent from the sine of 90° to the sine of 26° 1′ (= comp. of 63° 59′), will, on the tangents, reach from 25° 55′ to 12° 2′ : But the 4th arc is to be a cotangent; therefore 77° 58′ (the comp. 12° 2′) is the 4th arc.

The extent from the tangent of 29° 50′ to the tangent of 63° 59′, will reach on the line of sines from 12° 2′ (= comp, of 77° 58′) to 48° 9′.

But the 5th arc is to be a cosine; therefore 41° 51′ (the comp. of 48° 9′) is the fifth arc.

And 41° 51′ taken from 77° 58′ leaves 36° 7′ for the angle B.

The extent from the tangent of 29° 50′ to the tangent 63° 59′ is thus taken. Set one foot on the tangent 29° 50′, and extend the other to the tangent of 45°: Apply this extent on the tangents from 63° 59′ towards the left; rest the left hand foot, and extend the other to 45°, and the compasses will then have the required extent.

To find AC.

Say, as radius, to the cosine of the angle C.

So is the tangent of CB, to the tangent of a 4th arc.

And as cosine of CB, to the cosine of AB.

So is the cosine of the 4th arc, to the cosine of the 5th arc.

Then the difference between the 4th and 5th arcs will give the side AC.

The extent on the sines from 90° to 64° 5′ (the comp. of 25° 55′) will reach on the tangents from 63° 59′ towards the right to 61° 31′ the 4th arc.

Also the extent on the sines from 26° 1′ (= comp. of 63° 59′) to 60° 10′ (= comp. of 29° 50′) will reach from the sine of 28° 29′ (the complement of 61° 31′) to the sine of 70° 37′.

But

But the 5th arc is to be a cosine, therefore 19° 23′ is the 5th arc.

And 19° 23′ taken from 61° 31′ leaves 42° 8′ for the side AC.

SOLUTION *of* CASE II.

EXAM. In the spherical triangle ABD.

Given AD = 42° 9′
∠A = 127 50′
∠B = 36 8

Required the triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 2. Pl. VII.

1st. From B, with the secant of ∠ B, 36° 8′, cut the diameter ⊙ E in C; on C, with the same radius, describe the circumference B*a*F: then the angle DBF = the given ∠ B.

2d. Make the angle *n a q* equal to 37° 50′, the difference between 127° 50′ and 90°.

3d. Make *a q* = tangent of DA, 42° 9′; on ⊙ with the secant of 42° 9′ describe an arc *q*Q: on C with C *q*, cut the arc *q* Q in Q.

4th. Draw Q ⊙ G cutting the primitive circle in D, and BD will be a side of the triangle.

5th. From Q with Q*a*, cut B*a*F in A; and through D, A, G, describe a circumference, and the triangle BAD is that required. Whose parts BD, BA and ∠ D are thus measured.

6th. BD measured on the chords, gives 64 degrees.

7th. Make ⊙ P = tangent of half ∠ B, *viz.* 18° 4′; a ruler on P and A gives *x*; then B*x* measured on the chords gives 29° 50′, for BA.

8th. Draw a diameter perpendicular to GD, cutting the circumference DAG in *s*; a ruler on D and *s* gives *m*; make *mn* 90 degrees, then G *n* measured on the chords, gives 25° 55′ for the ∠ D.

II.

II. *To put* AB *on the primitive circle.* Fig. 2. 2.

1st. From A, with the secant of the supplement of the ∠A, *viz.* 52° 10′, cut the diameter ⊙F continued in C; on C, with the same radius, describe a circumference A*a*E.

2d. Make ⊙ P = the tangent of half the supplement of ∠ A, *viz.* 26° 5′; and make A *x* = chord of AD, 42° 9′: a ruler on P and *x*, gives D; then is AD equal to 42° 9′.

3d. On ⊙, with the tangent of the angle B, 36° 8′, describe an arc *mc*; on D, with the secant of ∠ B, 36° 8′, cut the arc *mc* in *c*; on *c*, with the same radius, describe a circumference DB, then the triangle ADB, will be that required.

III. *To put* DA *on the primitive circle.* Fig. 3. 2.

1st. Lay down AD with the chord of 42° 9′: Draw the diameter DF; and another ⊙H, perpendicular to DF.

2d. On A, with the secant of the supplement of ∠ A, *viz.* 52° 10′, cut the diameter E ⊙ in C; and on C, with the same radius, describe the circumference ABG.

3d. Make ⊙ P equal to the tangent of half the supplement ∠ A, *viz.* 26° 5′, a ruler by G and P gives *x*.

4th. Make *xm* = *xn* with the chord of ∠B, 36° 8′; a ruler by G and *n* gives *r*, by G and *m* gives *s*; on *b* the middle of *rs*, with the radius *bs*, cut ⊙ H in *p*.

5th. A ruler on F and *p*, gives *h*; make *hk* = *h*D; a ruler or F and *k* gives *c*; with the radius *c* D, describe the circumference DBF; and the triangle ABD, is that sought.

Computation by the Logarithmic Scales.

To find the side BD.

Say, as the sine of ∠ B, is to the side AD.
So is the sine of ∠ A, to the side BD.

Then the extent from the sine of 36° 8′ to the sine of 42° 9′, will reach from the sine of 52° 10′ (the supplement of 127° 50′) to the sine of 63° 59′ = side BD.

To find the side AB.

Say, as radius, is to the cosine of the ∠ A.
So is the tangent of AD, to the tangent of a 4th arc.
And, as tangent of ∠B, to the tangent of the ∠A.
So is the sine of the 4th arc, to the sine of a 5th arc.

Then the difference between the 4th and 5th arcs will be equal to the side AB.

The extent from the radius, or the sine of 90° to the sine of 37° 50′ (the complement of 52° 10), will reach on the tangents from 42° 9′ to 29° 02′ = 4th arc.

And the extent from the tangent of 36° 8′ to the tangent of 52° 10′, will reach on the sines from 29° 02′ to 58° 54′ = 5th arc.

Then the difference between 58° 54′ and 29° 02′ gives 29° 52′ for the side AB.

The extent from the tangent of 36° 8′ to the tangent of 52° 10′ is taken as shewed in the second operation of the first case.

To find the ∠D.

Say, as radius, is to the cosine of AD.
So is the tangent of ∠ A, to the tangent of a 4th arc.
And as the cosine of ∠ A, to the cosine of ∠ B;
So is the sine of the 4th arc, to the sine of the 5th arc.

Then

Then the difference between the 4th and 5th arcs will give the ∠D.

Now the extent from the ſine of 90° to the ſine of 47° 51′ (the complement of 42° 09′), will reach from the tangent of 52° .0′ to the tangent of 43° 40′. But the 4th arc being a cotangent will be 46° 20, the complement of 43° 40′.

Alſo the extent from the ſine of 37° 50′ (the complement of 52° 10′) to the ſine of 53° 52′ (the compliment of 36° 08′), will reach from the ſine of 46° 20′ to the ſine of 72° 15′ the 5th arc.

Then the difference between 72° 15′ and 46° 20′ *viz.* 25° 55′ will be the angle D.

In applying the firſt extent, *viz.* from the ſine of 90° to ſine of 47° 51′, to the tangents; ſet one foot on the tangent of 45° and let the other foot reſt where it falls; move the foot from 45° to 52° 10′; then this extent will reach from 45° to 43° 40′.

SOLUTION *of* CASE III.

Ex. In the ſpherical triangle ABD.

Given AB = 29° 50′.
BD = 63 59
∠B = 36 8
Required the triangle.

I. *To put* AB *on the primitive circle.* Fig 1. 3. Pl. VII.

1ſt. Make AB = chord of 29° 50′, draw the diameter BF, and another ⊙ E perpendicular thereto.

2d. From B, with the ſecant of ∠B, 36° 8′ cut ⊙ E in C, the center of BDF.

3d. From ⊙, with the tangent of half ∠ B, *viz.* 18° 4′, cut ⊙ E in P, the pole of BDF.

4th. Make B *x* = BD, 63° 59′; a ruler on P and *x*, gives D. Through A, D, G, deſcribe a circumference, and the triangle ADB is that required, whoſe parts AD, ∠ A, and ∠ D may be thus meaſured.

5th. A ruler on A and s gives z, make $z\,y$ = chord of 90°; a ruler on A and y gives p the pole of A s G; a ruler on p and D, gives n, and A n measured on the chords gives 42° 8′ for AD.

6th. G y measured on the chords, gives 52° 11′ for the supplement of ∠ A; therefore ∠ A = 127° 49′.

7th. A ruler on D and p gives r, on D and P, gives m; and rm, measured on the chords gives 25° 56′ for the angle BDA.

II. *To put* DB *on the primitive circle.* Fig. 2. 3.

1st. Make DB = chord of 63° 59′: draw the diameter BF and perpendicular thereto, the diameter ⊙ G.

2d. From B, with the secant of ∠ B, 36° 8′, cut ⊙ G in C; on C with CB, describe the circumference BAF.

3d Make ⊙ P = tangent of half ∠ B, 18° 4′, and D x = chord of AB 29° 50′, a ruler on P and x gives A; through D, A, E, describe a circumference, and the triangle ABD is that required.

III. *To put* AD *on the primitive circle.* Fig. 3. 3.

1. In a right line ed, touching the primitive circle in any point b, take bd = tangent of BD, 63° 59′; and be = tangent of AB, 29° 50′.

2. Make the angle dba = ∠ B, 36° 8′, and make $ba = be$.

3. From d, ⊙, with Da, ⊙e, describe arcs crossing in x; from x, d, draw the diameters AE, DF; and others OG, OH, perpendicular to AE, FD.

4. From d, x, with bd, eb, describe arcs crossing in B; and draw dB, xB.

5. From B draw BC, perpendicular to xB, and meeting ⊙G produced in C; also draw Bc perpendicular to dB, and meeting ⊙H in c; then C is the center of a circumference through A, B, E; and c the center of that through D, B, F; and the triangle ABD is that required.

Compu-

Computation by the Logarithmic Scales.

To find the angles A *and* C.

Say, as the sine of half the sum of the given sides
To the sign of half their difference;
So is the cotangent of half the given angle
To the tangent of half the difference of the required angles.

And, as the cosine of half the sum of the given sides
To the cosine of half their difference;
So is the cotangent of half the given angle
To the tangent of half the sum of the required angles.

Then the half difference of the required angles added to their half sum will give the greater angle A.

And the half difference of those angles taken from their half sum will give the lesser angle D.

Now the sum of the given sides 63° 59′ and 29° 50′ is 93° 49′, their difference is 34° 09′; the half sum = 46° 54½′, and the half difference is 17° 04½′.

Also half the given angle B is 18° 04′.

Then the extent from the sine of 46° 54′ to the sine of 17° 4′, will reach from the tangent of 71° 56′ (the complement of 18° 4′) to the tangent of 50° 57′ the half difference of the required angles.

Here the extent on the sines is from right to left or decreasing; so the extent on the tangents must be from left to right, which in this case is decreasing.

Also the extent from the sine of 43° 6′ (the complement of 46° 54′) to the sine of 72° 56′ (the complement of 17° 04′), will on the scale of tangents reach from 71° 56′ (the complement of 18° 4′) to 76° 53′ the half sum of the required angles.

Then the sum of 76° 53′ and 50° 57′ = 127° 50′ = ∠A.

And the difference of 76° 53′ and 50° 57′ = 25° 56′ = ∠C.

The angles being known, the other side may be found by opposite sides and angles, and is 42° 08′.

Or the other ſide may be found without knowing the angles.

Say, as radius is to the coſine of the given angle;

So is the tangent of either given ſide, to the tangent of a 4th arc.

Which 4th arc will be like the ſide uſed when the given angle is acute, otherwiſe it will be of a contrary kind with the ſide uſed.

Then take the difference between the 4th arc and the other given ſide, call the remainder a 5th arc.

And as the coſine of the 4th arc is to the coſine of a 5th arc;

So is the coſine of the ſide uſed in the former proportion

To the coſine of the ſide required.

Now the extent from the ſign of 90° to the ſine of 53° 52′ (= complement of 36° 08′) will reach from the tangent of 29° 50′ to the tangent of 24° 51′ the 4th arc.

And 24° 51′ taken from 63° 59′ leaves 39° 8′ for the 5th arc.

Then the extent from the ſine of 65° 09′ (the complement of 24° 51′) to the ſine of 50° 52′ (the complement of 39° 08′) will reach from the ſine of 60° 10′ (the complement of 29° 50′) to the ſine of 47° 51′; whoſe complement, *viz.* 42° 09′ is the ſide required.

Solution *of* CASE IV.

Ex. In the ſpherical triangle ABD:

Given ∠ D = 25° 55′.
∠ B = 36° 08′.
DB = 63° 59′,

Required, The triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 4. Pl. VII.

1. Make DB = chord of 63° 59′; draw the diameter BF, and draw ⊙G perpendicular to BF.

2. From

2 From B, with the secant of ∠ B, 36° 8′, cut ⊙G in C; and C will be the center of BAF.

3. From D, with the secant of ∠ D, 25° 55′; cut ⊙H in *c*, and *c* will be the center of DAE; and the triangle DAB is that which was required; whose parts DA, BA, and ∠A, are thus measured.

4. Make ⊙*p* = tangent of $\frac{1}{2}$ ∠D, 12° 57$\frac{1}{2}$′, a ruler on *p* and A gives *x*; then D*x* measured on the chords gives 42° 10′ for AD.

5. Make ⊙P = tangent of $\frac{1}{2}$ ∠B 18° 4′, a ruler on P and A, gives *z*; then BZ measured on the chords, gives 29° 54′ for AB.

6. A ruler on A and *p*, gives *n*, on A and P, gives *m*; and *nm* measured on the chords gives 52° 10′ the supplement of the angle A. Therefore ∠A = 127° 50′.

II. *To put* DA *on the primitive circle.* Fig. 2. 4.

1st. From D, with the secant of ∠D, 25° 55′; cut ⊙F in C; and C is the center of the circumference DBE.

2d. Make ⊙P = tangent of $\frac{1}{2}$ ∠D, 12° 57$\frac{1}{2}$; and make D*x* = chord of BD, 63° 59′; a ruler on P, *x*, gives B; and DB is 63° 59′.

3d. Make the angle CB*c* = ∠ B, 36° 8′; through C, draw *mc* perpendicular to B⊙, cutting B*c* in *c*; on *c*, with the radius *c*B, describe the circumference ABG; and the triangle ABD, is that which was required.

III. *To put* AB *on the primitive circle.* Fig. 3. 4.

1st. From B, with the secant of ∠B, 36° 8′ cut ⊙F in C; and C is the center of the circumference of BDE.

2d. Make B*x* = chord of BD, 63° 59′; and ⊙P = tangent of $\frac{1}{2}$ ∠ B, 18° 4′; a ruler on P and *x* gives D; then is BD = 63° 59′.

3d. Make the angle CD*c* = ∠D, 25° 55′; then *mc* drawn perpendicular to ⊙ D, meeting D*c* in *c*, gives *c* the center of the circumference ADG; and the triangle ABD will be that required.

Computation by the Logarithmic Scales.

To find the angle A.

Say, as radius is to the cosine of the given side;

So is the tangent of either given angle to the cotangent of a 4th arc.

Call the difference between the other given angle and the 4th arc, the 5th arc.

And, as the sine of the 4th arc, is to the sign of the 5th arc;

So is the cosine of the angle used in the former proportion

To the cosine of the required angle.

The 4th arc will be of the same kind with the angle first used if the given side is less than 90°; but of a contrary kind if that side is greater than 90°.

Arcs are said to be of the same kind, when both are less, or both greater, than 90 degrees.

The required angle will be of the same kind with the angle used in the proportions, if the 4th arc is less than the other angle; but of an unlike kind when the 4th arc is greater than the other angle.

Now the extent from the sine of 90° to the sine of 26° 01′ (the complement of 63° 59′) will reach from the tangent of 25° 55′ to the tangent of 12° 02′: But this is the complement of the 4th arc, which is 77° 58′.

And 36° 08′ taken from 77° 58′ leaves 41° 50′ for the 5th arc.

Then the extent from the sine of 77° 58′ to the sine of 41° 50′, will reach from the sine of 64° 5′ (the complement of 25° 55′) to the sine of 37° 50, which is the complement to 52° 10′.

But as the 4th arc was greater than 36° 08′, the angle sought is to be of a contrary kind to 25° 55′ (= ∠D), that is, that A is to be obtuse; so 127° 50′ (the supplement of 52° 10′) is to be taken for the angle A.

Now

Now all the angles and one side being known, the other sides may be found by the proportion subsisting between the sines of angles, and the sines of their opposite sides.

Or say,

As the sine of half the sum of the given angles
Is to the sine of half the difference of those angles;
So is the tangent of half the given side
To the tangent of half the difference of the required sides.

And

As the cosine of half the sum of the given angles
Is to the cosine of half the difference of those angles;
So is the tangent of half the given side
To the tangent of half the sum of the required sides.

Then the half difference added to the half sum gives the greater of the sought sides.

And the half difference subtracted from the half sum gives the lesser of the sought sides.

Now the half sum of the given angles, *viz.* $\frac{1}{2}\angle D + \frac{1}{2}\angle B = 31^\circ\ 01\frac{1}{2}'$,

And the half difference of those angles, *viz.* $\frac{1}{2}\angle B - \frac{1}{2}\angle D = 5^\circ\ 6\frac{1}{2}'$,

Also the half of the given side DB, is $31^\circ\ 59\frac{1}{2}'$.

Then the extent from the sine of $31^\circ\ 1'$, to the sine of $5^\circ\ 6'$;

Will reach from the tangent of $31^\circ\ 59'$, to the tangent of $6^\circ\ 3'$.

And the extent from the sine of $58^\circ\ 59$ ($=$ complement of $31^\circ\ 01'$) to the sine of $84^\circ\ 54'$ (the complement of $5^\circ\ 6'$), will reach from the tangent of $31^\circ\ 59'$ to the tangent of $35^\circ\ 58'$.

Then the sum of $35^\circ\ 58'$ and $6^\circ\ 3'$, *viz.* $42^\circ\ 01'$ $=$ AD.

And the difference of $35^\circ\ 58'$ and $6^\circ\ 3'$, *viz.* $29^\circ\ 55'$ $=$ AB.

SOLU-

Solution *of* CASE V.

Ex. In the spherical triangle ABD.

Given AB = 29° 50°
AD = 42 9
BD = 63 59

Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 5. Pl. VII.

1st. Make AB = chord of 29° 50′; draw the diameter BF.

2. Make A*n* = chord of AD, 42° 9′; and B*m* = chord of BD, 63° 59′.

3d. From *n*, with the tangent of AD, 42° 9′, cut EA produced in C; and from C, with that radius, describe the arc *nn*; from *m*, with the tangent of BD, 63° 59′, cut FB produced in *c*; and from *c*, with the radius c*m*, cut the arc *nn* in D.

4th. Through A, D, E; B, D, F, describe circumferences, and the triangle ADB is that which was required; whose angles A, B, D, are thus measured.

5th. A ruler on A and *a*, gives *x*; on B and *b*, gives *z*; make *xy*, *zv*, each 90°; a ruler on A and *y* gives P, in a radius perpendicular to AE; and a ruler on B and *v* gives *p*, in a radius perpendicular to BF.

6th. E*y* measured on the chords, gives 52° 12′ for the supplement of the ∠A; therefore ∠A = 127° 48′.

7th. F*v* measured on the chords, gives 36° 10′ for the angle B.

8th. A ruler on D and P gives *t*, and on D and *p* gives *s*; then *ts* measured on the chords, gives 25° 58′ for the angle D.

The sides AD, DB, are put on the primitive circle, by a construction so like the foregoing one, that it is needless to repeat it. See figures 2. 5. and 3. 5.

Com-

Computation by the Logarithmic Scales.

To find the angle A.

The sides including the angle A are AD = 42° 09′
And AB = 29 50
Their difference call x = 12 19
The side opposite the ∠A is BD = 63 59

Then the sum of BD and x is 76° 18′; the half sum is 38° 09′;

And the difference of BD and x is 51° 40′; the half difference is 25° 50′.

Now take the extent on the line of sines, from the half sum 38° 9′ to either of the containing sides, as to 29° 50′; apply this extent from the other containing side 42° 09′ towards the left, there let the foot rest, and extend the other point (*viz.* that which was set on 42° 09′) to the half difference 25° 50′; then this extent applied to the line of versed sines, will reach from 0 degrees (at the beginning) to 52° 12′; the supplement of which, or 127° 48′ will be the degrees in the angle A.

Again. *To find the angle* D.

The sides including the angle A, are BD = 63° 59′
And AD = 42 09
Their difference call x = 21 50
The side opposite to the ∠D is AB = 29 50

Then the sum of AB and x is 51° 40′; the half sum is 25° 50′.

And the difference of BA and x is 8° 0′; the half difference is 4° 00′.

Then the extent on the sines from 25° 50′ to 63° 59′ will reach from the sine of 42° 09′ to some point beyond 90°; therefore apply the extent between 25° 50′ and 63° 59′ from the sine of 90° downwards, let the point rest where it falls, and bring that point which was set on 90° to 42° 09′; then will the distance between

between the feet shew how far the first extent would reach beyond 90°: Now apply this extent on the sines from the point opposite to the middle 1 on the line of numbers, the other foot falling upwards to the right, let it rest there, and extend the other foot to the half difference 4° 0′: Then this extent applied to the versed sines, one foot being set on the point opposite the middle 1 on the line of numbers, the other foot will fall on 154° 5′; the supplement whereof, *viz.* 25° 55′ will be the angle D.

Solution *of* CASE VI.

Ex. In the spherical triangle ABD:

Given ∠A = 127° 50′.
∠B = 36° 8′.
∠D = 25° 55′.

Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 6. Pl. VII.

1st. From B, with the secant of ∠B, 36° 8′, cut ⊙F in C, and C will be the center of the circumference through B, D, E.

2d. From ⊙, with the tangent of 52° 10′ the supplement of ∠A, describe an arc *xc*.

3d. Make an angle C*aq* = ∠D, 25° 55′; make *aq* equal B*x*. (= secant of 52° 10′.)

4th. From C, with the radius C*q*, cut *xc* in *c*; From *c*, with the radius *qa*, describe a circumference ADG; and the triangle ABD, is that which was required: whose sides AB, BD, DA, are measured as follows.

5th. A ruler on B and *a* gives *d*, and on A and *b*, gives *f*; make *dg*, *fh*, each 90 degrees; a ruler on *g* and B gives P, and on *h* and A, gives *p*, in ⊙F, ⊙H, drawn perpendicular to BE, AG.

6th. A ruler on P and D gives *n*, and on *p* and D, gives *m*.

7th.

7th. Then BA, B*n*, A*m*, measured on the chords, gives 29° 50′; 63° 59′; 42° 9′; for the respective measures of BA, BD, AD.

The directions for this construction, may be easily applied to the putting either of the other sides on the primitive circle. Fig. 2. 6. and 3. 6. Pl. VII.

Computation by the Logarithmic Scales.

To find the side BD:

The angles including the side BD, are ∠B = 36° 08′
And ∠D = 25 55

Their difference call x = 10 13

The supplement of the ∠ opposite to BD is 52 10

The sum of the supplement of ∠A and x is 62° 23′; the half sum is 31° $11\frac{1}{2}$′.

The difference of the supplement of ∠A and x is 41° 57′; the half difference is 20° $58\frac{1}{2}$′.

Now on the sines, the extent from the half sum 31° $11\frac{1}{2}$′ to 25° 55′ will reach from 36° 08′ to a fourth sine; and the extent from that fourth sine to the sine of the half difference 20° $58\frac{1}{2}$′ will reach on the versed sines from the beginning to about 64° the side sought.

And in like manner may the other sides be found.

SECT. XIX.

Of the proportional Compasses.

THOSE compasses are called proportional, whose joint lies between the points terminating each leg; in such a manner, that when the compasses are opened, the legs form a cross.

SUCH compasses are either simple or compound.

SIMPLE

Simple proportional compaſſes, are ſuch, whoſe center is fixed: One pair of theſe, ſerve only for one proportion.

Thus, if a right line is to be divided into 2, 3, 4, 5, *&c.* equal parts; or the cord of $\frac{1}{3}$, $\frac{1}{5}$ $\frac{1}{7}$, *&c.* part of a circumference is to be taken; there muſt be as many of ſuch compaſſes, as there are diſtinct operations to be performed.

In each caſe, take the length of the right line, or of the radius of the circle, between the longer points of the legs; and the diſtance of the ſhorter points will be the part required.

Compound proportional compaſſes, are thoſe wherein the center is moveable; ſo that one pair of theſe will perform the office of ſeveral pairs of the ſimple ſort.

In the ſhanks of theſe compaſſes are grooves, wherein ſlides the center, which is made faſt by a nut and ſcrew.

On each ſide of theſe grooves, ſcales are placed; which may be of various ſorts, according to the fancy of the buyer: But the ſcales which the inſtrument-makers commonly put on theſe compaſſes, are only two, *viz.* lines and circles.

By the ſcale of lines, a right line may be divided into a number of equal parts, not exceeding the greateſt number on the ſcale; which is generally 12.

Exam. I. To divide a given right line, (ſuppoſe of $7\frac{1}{2}$ inches long,) into a propoſed number of equal parts. (as 11.)

Operation. Shut the compaſſes; unſcrew the button; move the ſlider until the line acroſs it, coincides with the 11th diviſion on the ſcale of lines; ſcrew the button faſt; open the compaſſes, until the given line can be received between the longer points of the legs; then will the diſtance of the ſhorter points

points be the 11th part of the given line, as required.

By the ſcale of circles, a regular polygon may be inſcribed in a given circle; provided the number of ſides in the polygon, do not exceed the numbers on the ſcale, which commonly proceed to 24.

Exam. II. To inſcribe in a circle of a known radius, (ſuppoſe 6 Inches) a regular polygon of 12 ſides?

Operation. Shut the compaſſes; unſcrew the button; ſlide the center until its mark coincides with the 12th diviſion on the ſcale of circles; ſcrew the button faſt; take the given radius between the longer points of the legs; then will the diſtance of the ſhorter points, be the ſide of the polygon required.

These ſcales are applicable to ſeveral other uſes beſide the foregoing ones, in the ſame manner, as the like lines on the ſector are.

From theſe operations it is evident, that the lengths of the longer and ſhorter legs, (reckoned from the center,) muſt always be proportional to the diſtance of their extremities.

Therefore, to divide a right line into 2, 3, 4, 5, 6, 7, 8, *&c.* equal parts; the lengths of each leg, from the center, will be expreſſed by the following ſeries, the whole length of the inſtrument being taken for unity.

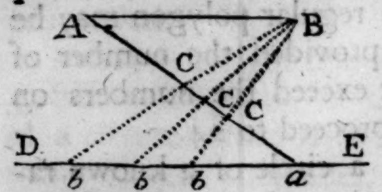
Longer leg $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, *&c.*
Shorter leg $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, *&c.*

These diviſions may be very accurately laid on the legs of the compaſſes by the help of a good ſector. (See Prob. 14.)

Or, the diviſions of this ſcale of lines may be found by the following conſtruction.

Draw

Draw the indefinite right line DE; and from any point A, without DE, draw Aa, equal to the shank



of the compasses, making any angle at a, with DE. Through A draw the right line AB, parallel to DE, and equal to the given line from whose parts the proportions are taken.

Let Aa contain N parts.

Now that ab may be the nth part of AB, or, that AB may be n times ab.

Let $ac = \frac{1}{n+1}$ N, or A$c = \frac{1}{n+1}$ N;

then the point c is the center of the screw pin. And through c, drawing BC, meeting DE in b; then is $ab = \frac{1}{n}$ of AB, or AB $= n$ times ab.

For $\frac{ab}{AB} = \frac{ac}{AC} = \frac{n}{1}$.

If the center of the screw-pin be distant from the mark in the slider, the $\frac{1}{m}$ part of N.

Then $ac = \frac{m \pm s}{s} \times \frac{N}{m}$ (putting $s = n + 1$.)

Ex. If N $= 10000$, $m = 400$, and $n = 1$, or 2, or 3, &c.

Then $ac = 5000$, or 3333, or 2500, &c. when the divisions on the shank respect the center pin.

And $ac = \left\{ \begin{matrix} 5025 \text{ or } 3358 \text{ or } 2525, \&c. \\ 4975 \text{ or } 3308 \text{ or } 2475, \&c. \end{matrix} \right\}$

when the divisions respect a mark in the slider, distant from the center pin, $\frac{1}{25}$ of the length of the Instrument.

The scale for dividing of the circle, or the divisions for regular polygons may be found thus.

Find the angles at the center, of as many regular polygons as are to be described on the compasses.

Seek the sines belonging to the half of each angle, to the radius 1.

To

To each of these sines doubled, add the radius 1.

THEN will the reciprocal of these numbers, be the lengths of the polygonal divisions, on the legs of the compasses, reckoned from the longer point; the length of the instrument being accounted unity.

FOR the longer and shorter legs, (or points) are in the same ratio, as are the radius and chord of the angle at the center.

AND as the sum of the radius and chord, is to the radius; so is the sum of the longer and shorter legs, (or points) to the length of the longer point.

AND hence was the following table composed, which shews the decimal parts on the leg, from the longer point to the center.

N° Sides.	Length on the Leg.	N° Sides.	Length on the Leg.	N° Sides.	Length on the Leg.
3	0,3333	11	0,6396	19	0,7523
4	0,4142	12	0,6589	20	0,7617
5	0,4597	13	0,6763	21	0,7706
6	0,5000	14	0,6921	22	0,7785
7	0,5354	15	0,7063	23	0,7860
8	0,5665	16	0,7193	24	0,7931
9	0,5940	17	0,7313		
10	0,6180	18	0,7423		

THESE divisions may be truly laid off by the help of a good sector; making the whole length of the proportional compasses, a transverse distance to 10 and 10, on the line of lines.

THE complements, to unity, of the numbers in the table, will give the distances of the divisions from the other point of the instrument.

If the mark in the ſlider, is at ſome diſtance from the center, as it commonly is, then this diſtance, which is always known, muſt be added to, or ſubtracted from, the foregoing numbers, according to that ſide of the center the mark is on; and the ſums, or remainders, will give the diſtances of the diviſions from one of the points.

About *Michaelmas*, 1746, was finiſhed a pair of proportional compaſſes, with the addition of a very curious and uſeful contrivance; (ſee the plate fronting the title page) *viz.* into one of the legs (A) at a ſmall diſtance from the end of the groove, was ſcrewed a little pillar *(a)* of about $\frac{1}{3}$ of an inch high, and perpendicular to the ſaid leg; through this pillar, and parallel to the leg, went a ſcrew pin *(bb)*; to one end of this ſcrew, was ſoldered a ſmall beam *(cc)* nearly of the length of the groove in the compaſſes; the beam was ſlit down the middle lengthwiſe, which received a nut (*f*) that ſlid along the ſlit (*dd*); this nut could be ſcrewed to the beam, faſt enough to prevent ſliding; one end (*e*) of the ſcrew of the nut (*f*) falls into a hole in the bottom of the ſcrew to the great nut (*g*) of the compaſſes; the ſcrew pin (*bb*) paſſed through an adjuſter (*h*): To uſe this inſtrument, ſhut the legs cloſe, ſlacken the ſcrews of the nuts *g* and *f*; move the nuts and ſlider *k* to the diviſion wanted, as near as can be readily done by the hand; ſcrew faſt the nut *f*; then by turning the adjuſter *h*, the mark on the ſlider *k*, may be brought exactly to the diviſion; ſcrew faſt the nut *g*; open the compaſſes; gently lift the end *e*, of the ſcrew of the nut *f*, out of the hole in the bottom of the nut *g*; move the beam round its pillar *a*, and ſlip the point *e*, into the hole in the pin *n*; ſlacken the ſcrew of the nut *f*; take the given line between the longer points of the compaſſes, and ſcrew faſt the nut *f*: Then may the ſhorter points of the compaſſes be uſed without any danger of the legs changing their poſition;

this

this being one of the inconveniencies that attended the proportional compaſſes before this ingenious contrivance; which was made by Mr. *Thomas Heath*, Mathematical Inſtrument-maker in the *Strand*, *London.*

THE proportional compaſſes had not been long invented before there were ſeveral learned and ingenious perſons who contrived a great variety of ſcales to be put thereon; but theſe are here omitted, becauſe the credit of the proportional compaſſes is greatly fallen, ſince the invention of the ſector, the latter being a much more uſeful inſtrument than the former, and not ſo ſubject to be put out of order; for if one of the points of theſe compaſſes ſhould be blunted or broke, the inſtrument cannot be uſed, until the damaged point be replaced by a new one. However, thoſe who are deſirous of knowing the conſtruction and uſe of ſuch ſcales on the proportional compaſſes, may be amply ſatisfied in conſulting *Hulſius*, *Horſcher*, *Galgemaire Bion*, and others mentioned in the preface to this book.

APPENDIX.

CONTAINING

The DESCRIPTION and USE of the

GUNNERS CALLIPERS.

A PAIR of Callipers is an inſtrument uſed to take the diameters of convex and concave bodies.

THE inſtrument called the Gunners Callipers, conſiſts of two thin rulers or plates, which are moveable quite round a joint, by the plates folding one over the other.

THE length of each ruler or plate is uſually between the limits of ſix inches and a foot, reckoned from the centre of the joint; and from one to two inches broad: But the moſt convenient uſeful ſize is about nine inches long. The figure is beſt ſeen in the plate.

ON theſe rulers are a variety of ſcales, tables, proportions, &c. ſuch as are eſteemed uſeful to be known by gunners and other perſons employed about artillery: But except the taking of the calibre of ſhot and cannon, and the meaſuring of the magnitude of *ſaliant* and *entring angles*, there are none of the articles with which the callipers are uſually filled, eſſential to this inſtrument; the ſcales are, or may be, put on the ſector; and the tables, precepts, &c. may be put into a pocket book, where they will not need ſo much contraction: However, for the ſake of thoſe who are deſirous

desirous of having a single instrument perform many things, the following articles and their disposition on the callipers are here offered: Some of which were proposed many years ago by my much esteemed friend Mr. *Charles Labelye*, engineer to the works of *Westminster-bridge*.

Articles proposed to be put on the Gunners Callipers.

I. THE measures of convex diameters in inches.

II. THE measures of concave diameters in inches.

III. THE weights of iron shot from given diameters.

IV. THE weight of iron shot proper to given gun bores.

V. THE degrees of a semicircle.

VI. THE proportion of Troy and Averdupoise weight.

VII. THE proportion of English and French feet and pounds.

VIII. FACTORS useful in circular and spherical figures.

IX. TABLES of the specific gravity and weights of bodies.

X. TABLES of the quantity of powder necessary for proof and service of brass and iron guns.

XI. RULES for computing the number of shot or shells in a finished pile.

XII. RULES concerning the fall of heavy bodies.

XIII. RULES for the raising of water.

XIV. THE rules for shooting with cannon or mortars.

XV. A LINE of inches.

XVI. LOGARITHMIC scales of numbers, sines, versed sines and tangents.

XVII. A SECTORAL line of equal parts, or the line of lines.

XVIII. A SECTORAL line of plans or superficies.

XIX. A SECTORAL line of solids.

The Callipers proposed for the reception of the foregoing articles is nine inches long, and each leg two inches broad at the head, and at the points; part of the breadth between the ends is hollowed away in a curve, in order to contain the curvature of the ball, whose diameter is taken between the points; one of ten inches diameter is the largest that can conveniently be taken with a nine inch Calliper; those of six inches cannot well be applied to a shot of more than seven inches diameter.

For the ease of reference; it will be convenient to distinguish the four faces of the Callipers by the letters A, B, C, D: Each of the faces A and D consist of a circular head and a leg; the other faces B and C consist only of a leg.

ARTICLE I.

Of the measures of convex diameters.

On part of the circular head joining to the leg of the face A, are divisions distinguished by the title of *shot diameters*: These are to shew the distance in inches, and tenth parts of an inch, of the points of the Callipers when they are opened.

THE USE.

Open the points of the Callipers so, that they may take in the greatest diameter of the ball; then will the bevil edge marked E shew among the foresaid divisions, the diameter of that ball in inches and tenth parts, not exceeding ten inches.

These divisions may be thus laid down by the sector.

Open the sector until the radius of the circle, whereon is marked the scale of divisions on the head of the Callipers, taken with the compasses, falls transversely in the scales of lines, on the divisions shewing the distance between the centre of the Callipers and its points: Then the transverse distances of the several divisions

diviſions on the ſcales of lines, being applied like chords to the circle of diviſions on the head of the Callipers will give the diviſions required.

Thus in the nine inch Callipers, the radius of the head, or circle of diviſions being one inch, and the breadth at the points two inches; the diſtance between the centre and points will be ($\sqrt{82}$ =) 9,055385: Then one inch being made a tranſverſe diſtance on the ſcales of lines t[illegible]o $\frac{5}{100}$; the tranſverſe diſtances of 10, 9, 8, 7, 6, &c. being applied to the circle on the head of the Cal[illegible]s appropriated for the ſcale, from the mark where the diviſions commence, will give the ſeveral points, which being cut by the bevil edge E will ſhew how far the points of the Callipers are diſtant.

The workmen generally lay theſe diviſions down by trial.

ARTICLE II.

Of the weights of iron ſhot.

On the circular bevil part E of the face B, is a ſcale of diviſions denominated by ℔ *weight of ſhot.* Theſe are to ſhew the weights of iron ſhot when the diameter is taken between the points of the Callipers: For then the number cut by the inner edge of the leg A, ſhews the weight of that iron ſhot in pounds averdupoiſe, when the weight is among the following ones, *viz.*

℔. $\frac{1}{2}$. 1. $1\frac{1}{2}$. 2. 3. 4. $5\frac{1}{4}$. 6. 8. 9. 12. 16. 18. 24. 26. 32. 36. 42.

Observing that the figures neareſt the bevil edge anſwer to the ſhort lines; and thoſe figures behind them anſwer to the diviſions marked with the longer ſtrokes or lines.

These diviſions are to be laid down from a table ſhewing the diameters of iron ſhots to given weights. Such a table is computed by knowing the weight of one ſhot of a given diameter: Thus an iron ſhot of

four inches diameter is found to weigh nine pounds: Then the weights of ſpheres being to one another as the cubes of their diameters, Say, As 9 ℔ is to 64, the cube of 4.

So is any other weight, to the cube of its diameter.

THEN the cube root being taken will give the diameter.

Now ſetting the points of the Callipers to touch one another, make a mark on the bevil edge E oppoſite to the inner edge of the leg A; and this mark will be the beginning of this ſcale of weights: The other diviſions will be obtained by opening the points of the Callipers to the diſtances reſpecting the weights to be introduced, as ſhewn by the table, and marking the diviſion oppoſite to the inner edge of the leg A.

ARTICLE III.

Of the meaſures of concave diameters.

ON the lower part of the circular head of the face A, and to the right hand of the diviſions for the diameters of ſhot, is another ſcale of diviſions, againſt which ſtands the words *Bores of Guns.*

THE USE.

To find the calibre, or the diameter of the bore of a cannon.

SLIP the legs of the Callipers acroſs each other, until the ſteel points touch the concave ſurface of the gun in its greateſt breadth; then will the bevil edge F, of the face B, cut a diviſion in the ſcale ſhewing the diameter of that bore in inches and tenth parts.

IN the nine inch Callipers theſe diviſions may be extended to 18 inches diameter, but 14 inches is ſufficient for both cannon and mortars: And in the ſix inch Callipers a diameter greater than 10 inches cannot be conveniently introduced.

Theſe

These divisions may be thus laid down by the sector.

SET one inch the radius of the circle on which the divisions are to be put, as a transverse distance to the division $9\frac{5}{100}$ on the scale of lines on the sector: Set the points of the Callipers together, and make a mark on the circular head where it is then cut by the bevil ege F: Then the several transverse distances taken from the sector, and applied on the circumference of the circular head of the Callipers, from the said mark, the several divisions shewing the distance of the points of the Calliper are thereby obtained.

WORKMEN find these divisions by actually setting the points to the distance.

ARTICLE IV.

Of the weights of shots to given gun bores.

WITHIN the scales of shot and bore diameters on the circular part of the face A are divisions marked *Pounders.*

THE USE.

WHEN the bore of a gun is taken between the points of the Callipers, the bevil edge F will cut one of these divisions, or be very near one of them: Then the number standing against it will shew the weight of iron shot proper for that gun; not exceeding 42 pounds.

THE inner figures $\frac{1}{2}$, $1\frac{1}{2}$, 3, $5\frac{1}{4}$, 8. 12. 18. 26. 36. belong to the longest strokes or lines; and the figures 1. 2. 4. 6. 9. 16. 24. 32. 42 belong to the short strokes.

THE diameters given by these pounders are larger than those given for the same weights of shot; because there is an allowance made, *called Windage*, that the shot may roll easily along the chace.

AR-

ARTICLE V.

Of the degrees in the ſemicircular head.

THESE degrees are placed on the upper half of the circular head of the face A, where are three concentric ſcales of degrees: The outward ſcale has 180 degrees numbered from the right to left, with 10. 20. 30. 40. &c. to 180: The middle ſcale is numbered in the ſame manner, but the contrary way: And the inmoſt ſcale begins in the middle with 0, and is numbered from thence both ways with 10. 20. 30. &c. to 90 degrees.

THE USE.

Firſt to meaſure an entring, or internal, angle.

APPLY the legs of the Callipers ſo that its outſide edges coincide with the legs of the given angle; then will the bevil edge F cut the degrees ſhewing the meaſure of that angle in the outſide ſcale

Secondly. To meaſure a ſaliant, or external, angle.

SLIP the legs of the Callipers acroſs each other, ſo as their outſide edges may coincide with the legs of the given angle; then will the bevil edge E cut the degrees ſhewing the meaſure of that angle: Theſe degrees are to be counted on the middle ſcale.

HENCE an angle of any number of degrees may be readily laid down by the Callipers, either on paper, or in the field.

Thus. OPEN the Callipers, the legs being croſſed, until the edge E cuts the degrees on the middle ſcale; the croſſing edges of the inſtrument will then form the ſides of that required angle: The Callipers then laid flat on the paper or ground, lines drawn by the ſtrait ſides will expreſs that angle.

Thirdly.

Thirdly. To find the elevation of cannon and mortars, or of any other oblique plane or line.

PASS one end of a fine thread into the notch on the plate B, and to the other end tie a bullet, or other weight: Apply the ſtrait ſide of the plate A to the ſide of the body whoſe inclination is wanted; hold the plate A in this poſition, and move the plate B until the thread falls upon the line near the centre marked *Perp.* Then will the bevil edge F cut the degrees, counted on the inner ſcale, ſhewing the inclination which that body makes with the horizon.

Note. When the edge F cuts o on the inner ſcale; and the ſtring cuts the *Perp.* mark, then the ſtrait ſide of the leg A is horizontal: If the head of the Callipers is elevated above the other end, then the edge F muſt ſlide downwards towards the ſtrait ſide of the leg A: But if the head of the Callipers is held lower than the other end, then muſt the edge F ſlide the contrary way.

As the outſide of a cannon or mortar is not parallel to its chaſe; therefore a ſtrait ſtick muſt be applied to the bottom or top of the bore, touching the chaſe; and the ſide of the Callipers be laid on that ſtick.

ARTICLE VI.

Of the proportion of Troy and Averdupoiſe Weights.

ON the face C near the point of the Callipers is a little table ſhewing the number of pounds that are contained in an equal weight expreſſed in pounds Troy; and the contrary.

THESE numbers are taken from very accurate experiments made in the year 1744 by the late *Martin Folkes*, Eſq; Preſident of the Royal Society, aſſiſted by ſeveral other gentlemen of that learned Body.

THE

THE TABLE.

℔ Troy	℔ Averd.	oz. Troy	oz. Averd.
576,00000	=700	82	=90
1,00000	= 0,82274	1,00000	= 1,09707
1,21545	= 1,00000	0,91152	= 1,00000

THE USE.

EXAMPLE I. *What weight in pounds Troy is equal to a brass gun weighing 18 C wt.*

Now 18 C wt. is equal to 2016 ℔ (=18×112).
THEN 1 : 1,21545 : : 2016 : 2450 ℔ Troy.
OR, 0,82274 : 1 : : 2016 : 2450 ℔ Troy.
OR, 576 : 700 : : 2016 : 2450 ℔ Troy.
EITHER of these methods may be used as the operator pleases.

EXAMPLE II. *What is the worth of a ton of gold; supposing 1 ℔ Troy makes 44½ guineas.*

Now 1 Ton =2240 ℔ Averd. (=20×112).
AND 1 : 1,21545 : : 2240 : 2722,6 ℔ Troy.
ALSO 44½ Guineas, makes 46,725 £ sterling.
THEN 1 : 2722,6 : : 46,725 : 127213.485 £.
OR, 127213 *l.* 9*s.* 8*d.* ½.
BUT if Troy pounds were given to be converted into Averdupoise pounds, then the numbers in the Troy column must be the first terms of the proportions.

EXAMPLE III. *If a brass gun weighs 2450 ℔ Troy; What is its weight in Averdupoise?*

THEN 1 : 0,82274 : : 2450 : 2015,7 ℔ Ave.
OR, 1,21545 : 1,00000 : : 2450 : 2015,7
OR, 700 : 576 : : 2450 : 2016.

ALTHOUGH the Averdupoise pound is heavier than the Troy pound, yet the Troy ounce is heavier than the Averdupoise ounce, nearly in the proportion of 90 to 82.

Ex-

EXAMPLE IV. *In a cheſt of ſilver containing 4380 pieces of eight, each piece weighing $\frac{4}{5}$ of an ounce Troy: How many ounces Averdupoiſe.*

THEN	82	:	90	: :	$4380 \times \frac{4}{5}$	: 3845,88
Or,	1	:	1,09707	: :	$4380 \times \frac{4}{5}$	: 3844,13
OR,	0,91152	:	1	: :	$4380 \times \frac{4}{5}$	: 3844,13

CLOSE to the former table is another, ſhewing the number of cubic inches in a gallon, both in wine and beer meaſures; and conſequently their proportions: One uſe is ſhewn by the following Example.

How long will 33 butts of beer ſerve a crew of 324 men, allowing to each man 3 wine quarts a day?

NOW 33 butts contain 3564 beer gall. $(=108 \times 33)$
AND 231 : 282 : : 3564 : $4350\frac{6}{7}$ wine gallons,
AND $4350\frac{6}{7}$ gallons makes $17403\frac{3}{7}$ quarts,
THEN 17403 divided by 324 gives very near 54.
CONSEQUENTLY $\frac{1}{3}$ of 54, or 18 days, is the time that the beer will ſerve.

IF wine gallons were to be converted into beer gallons,

SAY 282 : 231 : : wine gallons : beer gallons.
OR 94 : 77 : : W. G. : B. G.

ARTICLE VII.

Of the proportion of the Engliſh and French feet and pounds.

NEAR the point of the face D of the Callipers are two tables ſhewing the proportion between the pound weights of *London* and *Paris*, and alſo between the lengths of the foot meaſure of *England* and *France*. Theſe are according to the accurate ſtandards ſettled between the Royal Societies of *London* and *Paris* about the year 1743.

THE

THE TABLES.

Eng. ℔.		F. ℔.	Eng. F^t.		Fr. F^t.
1,08	=	1,00	114	=	107
1,00	=	0,926	1,000	=	0,9386
108	=	100.	1,0654	=	1,0000

THE USE.

EXAMPLE I. *Suppose a crew of 54 English sailors were to attack a French fort, and carry off 6 pieces of brass cannon weighing one with another 980 ℔ French: How much would each John's share come to, supposing they could sell the cannon at 8 l. a hundred weight English?*

	℔ F.		℔ E.		℔ F.		
Now	100	:	108	: :	980×6	:	6350,4, ℔ Engl.
	℔		£.		£.		
AND	112	:	8	: :	6350,4	:	453,6 £. sterling.
	M.		£.		M.		
THEN	54	:	453,6	: :	1	:	8,4 £.

So that the share of each will be 8 guineas.

EXAMPLE II. *How many English yards are equal to 180 French toises or fathoms?*

Now 1 : 1,0654 : : 180 : 191,672 Eng. Fa.

THEN 180 French Fathoms are equal to about 383 yards 1 foot.

ARTICLE VIII.

Factors useful in circular and spherical figures.

NEAR the point of the Callipers on the face A is a table containing four rules of the circle and sphere.

THE

THE TABLE.

Diam.	×3,1416	=circumf.	of a circle.
Sq. Diam.	×0,7854	=area	
Sq. Diam.	×3,1416	=ſurface	of a ſphere.
Cube Diam.	×0,5236	=ſolidity	

THE USE.

EXAMPLE I. *What is the circumference of a circle whoſe diameter is 12 inches?*

THEN (3,1416×12 =) 37,6992 is the circumfer.

EXAMPLE II. *What is the area of a circle whoſe diameter is 12 inches?*

Now the ſquare of 12 is 144.
THEN (0,7854×144 =) 113,0976 is the area.

EXAMPLE III. *What is the ſuperficies of a ſphere whoſe diameter is 12 inches?*

Now the ſquare of 12 is 144.
THEN (3,1416×144=) 452,3904 the ſuperficies of the ſphere.

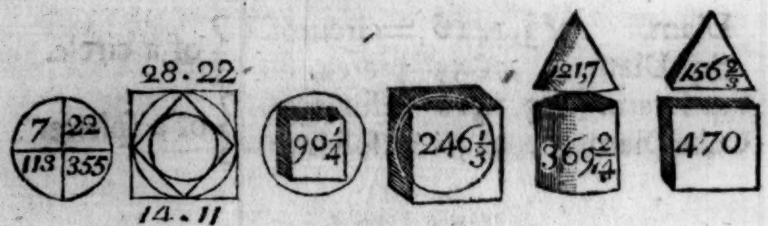
EXAMPLE IV. *Required the ſolidity of a ſphere whoſe diameter is 12 inches?*

Now the cube of 12 is 1728.
THEN (0,5236×1728=) 904,7808 is the ſolidity.

Upon the circular heads of Callipers are uſually placed certain mathematical figures with numbers ſet to them; which figures and their numbers may be placed near the points of the Callipers here deſcribed, the circular head being appropriated for another uſe.

The

The figures are these.



1 2 3 4 5 6

THE numbers in figure 1, are useful for finding the circumference of a circle by knowing its diameter; or to find the diameter by knowing the circumference. Thus

SAY As 7 : 22 : : any given diam: its circum.
AND As 22 : 7 : : any given circum: its diam.
OR As 113 : 355 : : any given diam: its circum.
AND As 355 : 113 : : any given circum: its diam.

FIG. 2. There is a circle inscribed in a square; a square within that circle, and a circle within the inner square: To this figure are set the numbers 28. 22. 14. 11. These numbers signify, that if the area of the outward square is 28, the area of the inscribed circle is 22; the area of the square inscribed in that circle is 14, and the area of its inscribed circle is 11.

THE USE.

EXAMPLE. *What is the area of a circle whose diameter is 12?*

Now the square of 12 is 144.
THEN As 28 : 22 : : 144 : 113,1 the area.
Or As 14 : 11 : : 144 : 113,1.

IT may be observed, that the squares are in proportion to one another as 2 to 1; and the two circles are also in the same proportion.

Figure 3. Represents a cube inscribed in a sphere; the number $90\frac{1}{4}$ fixed to it shews, that a cube of iron, inscribed

inſcribed in a ſphere of 12 inches in diameter, weighs $90\frac{1}{4}$ pounds weight.

Figure 4. Is to expreſs a ſphere inſcribed in a cube: Now this figure with its number $246\frac{1}{4}$ is to ſhew the weight in pounds of an iron ſphere of 12 inches diameter; or of a ſphere inſcribed in a cube whoſe ſide is 12 inches.

Figure 5. Repreſents a cylinder and cone, whoſe diameters and heights are each one foot: To the cylinder is annexed the number $369\frac{3}{14}$ ſhewing the weight in pounds of an iron cylinder of 12 inches diameter and 12 inches in height: And the number $121\frac{7}{100}$ joined to the cone ſhews that an iron cone the diameter of whoſe baſe is 12 inches, and the height 12 inches, weighs $121\frac{7}{100}$ pounds.

Figure 6. Shews that an iron cube, whoſe ſide is 12 inches, weighs 470 pounds; and that a ſquare pyramid of iron, whoſe baſe is a ſquare foot, and its height 12 inches, weighs $156\frac{2}{3}$ pounds.

THESE numbers which have hitherto been fixed to the four laſt figures are not ſtrictly true.

FOR by experiment an iron ſhot of four inches diameter weighs 9 pounds.

AND the weights of ſpheres being to one another as th ecubes of their diameters:

THEREFORE $64\ (=4\times4\times4) : 9 :: 1728\ (=12\times12\times12) : 243$ pounds, for the weight of a ſphere of iron which is 12 inches in diameter: Conſequently the number 243 ſhould be uſed inſtead of $246\frac{1}{4}$ in the 4th figure.

AGAIN. The ſolidity of a cube inſcribed in a ſphere of 12 inches in diameter, is 332,55 cubic inches.

AND the weights of bodies of a like matter being in the proportion of their ſolidities.

THEREFORE, As $904{,}7808 : 243 :: 332{,}55 : 89{,}315$ pounds.

CONSEQUENTLY the number $90\frac{1}{4}$ uſed at figure 3, ſhould be $89\frac{1}{3}$.

Here 904,7808 is the ſolidity of a ſphere of 12 inches diameter.

At figure 5. the weight of the iron cylinder ſhould be 364,5 inſtead of 369 $\frac{3}{14}$, and the weight of the cone ſhould be 121,5.

For the ſolidity of a cylinder of 12 inches diameter, and 12 inches high, is 1357,1712 cubic inches.

Then 904,7808 : 243 : : 1357,1712 : 364,5 pounds.

And cylinders and cones having equal baſes and heights are in proportion as 3 to 1.

Therefore the $\frac{1}{3}$ of 364,5, or 121,5 pounds is the weight of the cone.

The numbers at figure 6 annexed to the cube ſhould be 464 pounds.

And that fixed to the pyramid ſhould be 154$\frac{2}{3}$ pounds.

For the cube inches in a foot cube are 1728.

Then 904,7808 : 243 : : 1728 : 464.

And a pyramid is $\frac{1}{3}$ of a cube, the baſes and height being equal.

Therefore the $\frac{1}{3}$ of 464 is 154 $\frac{2}{3}$ pounds for the weight of the pyramid.

Although it is uſually reckoned that a four inch iron ſhot weighs nine pounds; and from thence it is deduced that the cubic foot weighs 464 pounds; yet by the table of ſpecific gravity on the callipers, which is framed from the moſt accurate experiments, a cubic foot of caſt iron weighs almoſt 446 pounds; which is 18 pound leſs than the weight derived from the 4 inch ſhot, and 24 pound leſs than that heretofore graved on the callipers; therefore all the weights found from the ſaid 4 inch ſhot, ſhould be diminiſhed in the proportion of 464 to 446.

For the numbers at figures 3, 4, 5, 6.

As 464 : 446 : : 89,315 : 85,85.
As 464 : 446 : : 243 : 233,5.
As 464 : 446 : : 364,5 : 350,3.

So

So $85\frac{4}{5}$ ℔ is the weight of an iron cube inſcribed in a ſphere of 12 inches in diameter.

And $233\frac{1}{2}$ ℔ is the weight of an iron ſphere of 12 inches diameter.

Also $350\frac{1}{3}$ ℔ is the weight of an iron cylinder of a foot in diameter and height.

And $116\frac{2}{3}$ ℔ is the weight of an iron cone of a foot in diameter and height.

Again 446 ℔ is the weight of a cubic foot of iron.

And $148\frac{2}{3}$ ℔ is the weight of an iron pyramid, having its baſe a ſquare foot, and its height equal to 12 inches.

ARTICLE IX.

Of the ſpecific gravities and weights of bodies.

On the leg B of the callipers is a table ſhewing the weights of a cubic inch or foot of various bodies in pounds averdupoiſe. To the table here annexed is joined the ſpecific gravities of thoſe bodies, which are omitted on the callipers for want of room.

A Table shewing the weights of bodies and their specific gravities.

Bodies.		Weights.	Spe. Gravity.
Fine Gold. Inch		0,710359	19,640
Standard Gold. Inch		0,706018	19,520
Quickfilver. Inch.		0,497657	13,762
Lead.	Foot	707,0458	11,313
	Inch	0,409170	
Fine Silver. Inch		0,401150	11,091
Standard Silver. Inch		0,384440	10,629
Copper	Foot	548,0628	8,769
	Inch	0,317166	
Brafs. F.		506,2746	8,104
Steel. F.		490,6241	7,850
Bar Iron. F.		485,2500	7,764
Block Tin. F.		452,3731	7,238
Caft Iron. F.		445,9363	7,135
White Marble. F.		168,8757	2,702
Glafs. F.		162,4994	2,600
Stone	Flint. F.	161,3745	2,582
	Portland. F.	160,6245	2,570
	Free. F.	158,2485	2,352
Brick. F.		125,0000	2,000
Brimftone. F.		112,5000	1,800
Clay. F.		112,0000	1,792
River Sand. F.		110,0000	1,760
Sea Water. F.		64,3732	1,030
Rain Water	Cubic F.	62,5000	1,000
	Cubic Inch	0,036169	
	Cylindric F.	49,080000	
	Cylindric Inch	0,028403	
Port Wine. F.		61,8000	0,988
Brandy. F.		58,0000	0,928
Olive Oil. F.		57,0624	0,913
Dry Oak. F.		57,1875	0,915
Lime. F.		52,0000	0,832
Elm and Afh. F.		50,0000	0,800
Wheat. F.		50,0000	0,800
Yellow Fir. F.		41,0625	0,657
White Deal. F.		35,5624	0,569
Gun Powder	F.	69,1200	1,106
	In.	0,0400	

IN

In the foregoing table is contained ſuch bodies as practical engineers and others may have occaſion to know their reſpective weights; there are indeed a great number of other bodies whoſe ſpecific gravity have been determined by ingenious men: But thoſe only which were apprehended to be the moſt uſeful were ſelected for this ſubject.

Every one will readily conceive how the column of weights may be obtained, namely by procuring maſſes of a cubic inch or foot of the ſolids, and carefully weighing them in nice ſcales to the ſmalleſt degree of averdupoiſe weight: And for the fluids, their weights may be determined by having cubical or cylindrical veſſels made to hold a known quantity of cubical inches, and in them to weigh the fluids.

The ſpecific gravity of a body being the relation which that body has to ſome other body fixed upon as a ſtandard to compare by; and rain water being found to be alike, or very nearly ſo, in all places; and therefore choſen by philoſophers as the proper ſtandard; conſequently by the word ſpecific gravity of a body is meant no more, than that it is ſo many times heavier or lighter than water, when compared together in equal balks.

Thus fine ſilver is ſomething more than 11; that is, a maſs of fine ſilver will weigh ſomething above eleven times the weight of an equal maſs of water: And, ſo a common brick weighs twice as much as the rain water that would fill a mould fitted to the brick.

Now the weights of equal maſſes of ſeveral bodies being determined, their ſpecific gravities may be readily found, they being in the ſame proportion to one another as their weights: And as the compariſon is made to rain water, of which, by repeated experiments, it has been found that a cubic foot weighed $62\frac{1}{2}$ pounds averdupoiſe; therefore *dividing the weight of a cubic foot of any body*, *by* $62\frac{1}{2}$; *the quotient will be*

the ſpecific gravity of that body, relative to rain water whoſe ſpecific gravity is repreſented by unity.

The difficulty of procuring maſſes of metals and other bodies in all parts homogenous, and of having both them and the veſſels of capacity conſtructed to a mathematical exactneſs, has rendered this method of eſtimating the ſpecific gravities from the weights of equal bulks, liable to exception: And therefore another method has been contrived to come at theſe ſpecific gravities, hydroſtatically.

It is a well known thing that any body will weigh leſs when it is immerſed in water than when it is weighed in the open air; and from a very little reflection, it will be ſeen that the difference between the weights of any body when weighed in air and in water, will be equal to the weight of ſo much water as is equal in bulk to the body immerſed: But the difference between the weights of a body in air and in water, will ſhew the weight of a bulk of water equal to the body ſo weighed: *Therefore to find the ſpecific gravity of any body, find its weight in air and in rain water, and take the difference of thoſe weights; then the weight in air divided by that difference, will give the ſpecific gravity required.*

If the ſolid whoſe ſpecific gravity is wanted, be lighter than water, ſo that it cannot ſink by its own weight, let it be joined to another ſo weighty that the compound may ſink: But firſt let the loſs of weight which the heavy body alone ſuſtains in water be found as before; and then let the loſs of weight which the compound body ſuſtains be diſcovered; from which take the loſs of weight of the heavier, and the remainder is the loſs of weight ſuſtained by the lighter; by which dividing the weight in air of the lighter body, and the quotient will ſhew the ſpecific gravity.

When the ſpecific gravity of fluids are to be compared to each other; take a ſolid of any matter and ſhape, ſuppoſe a glaſs ball, hung by a horſe hair, and

immerſe this ſolid in each fluid, and find the loſs of weight of the ſolid in each fluid, the weight of the body in air being firſt known; then will theſe loſſes expreſs the ſpecific gravities of thoſe fluids: For ſince the loſs of weight in each liquor is equal to the weight of as much of the liquor as is equal in bulk to the body weighed; therefore by taking the loſſes of weight ſuſtained by the ſame body in the ſeveral liquors, the abſolute weights are obtained of ſuch portions as are equal in bulk, and conſequently the ſpecific gravities of thoſe liquors.

In this method of finding the ſpecific gravity of ſolids, it is not neceſſary that they ſhould be reduced to any regular ſhape; neither is there wanted a veſſel of a known figure and capacity to contain the fluids; and conſequently the ſpecific gravities of bodies, whether ſolids or fluids, may be very eaſily come at: But from the ſpecific gravities to find the abſolute weights of any aſſigned maſs of ſeveral bodies, there muſt be another experiment made, which is to find the loſs of weight in water, of a body of a known magnitude; ſuppoſe of a cylinder of a homogenous metal, the ſolidity of that cylinder being moſt accurately calculated; then will the abſolute weight of an equal maſs of water be known; and conſequently the weight of a cubit foot of water may be accurately obtained, *from whence the abſolute weight of a cubic foot of any other body whoſe ſpecific gravity is known, may be found by multiplying the ſpecific gravity of that body by the weight of a cubic foot of water.*

SOME USES OF THE TABLE.

The weights of bodies anſwering to a given ſolidity are of a twofold uſe.

First, *To find the weight of a body of a given dimenſions, or ſolidity.*

SECONDLY, *To find the solidity of a body by knowing its weight.*

EXAM. I. *What is the weight of a block of marble 7 feet long, 3 feet broad, and 2 feet thick?*

Now $7 \times 3 \times 2 = 42$ feet for the solidity.

A CUBIC foot of marble weigh 168,8757 pounds.

THEN $168,8757 \times 42$ gives 7092,7794 pounds.

C qrs. ℔

OR, 63 : 1 : $8\frac{3}{4}$ is the weight of that marble.

EXAM. II. *What is the weight of a 13 inch iron bomb shell, the metal being two inches thick on a mean?*

HERE the solidity of two spheres must be found, one of 13 inches diameter, and the other of 9 inches diameter; then their difference being taken will give the solidity of the shell.

Now the cube of 13 is 2197.

AND the cube of 9 is 729.

ALSO $2197 \times 0,5236$ gives 1150,3492 solidity.

AND $729 \times 0,5236$ gives 381,7044 solidity.

THEIR difference is 768,6448 cubic inches.

AND 768,6448 divided by 1728 gives 0,4448 parts of a cubic foot.

Now a cubic foot of cast iron weighs 445,9363 pounds.

THEN $445,9363 \times 0,4448$ gives 198,363 pounds for the weight of the shell.

EXAM. III. *How many pigs each of 12 inches long, 6 wide and 4 thick, may be cast out of 10 ton of melted lead?*

Now 10 ton $= 10 \times 20 = 200$ C. wt.

AND $112 \times 200 = 22400$ pounds in 10 ton.

BY the table, 707,0458 pound makes a cubic foot of lead.

AND 22400 divided by 707,0458, gives 31,681 cubic feet, which the 10 ton will make.

Now

Now the ſolidity of each pig is $\frac{1}{6}$ of a foot.

THEREFORE 31,681 feet ſolid will make 190 pigs.

FROM ſeveral experiments it appears that middling ſized men, or thoſe between 5 feet 6 inches and 5 feet 9 inches in height, weigh about 150 pounds, and are in bulk equal to about $2\frac{3}{4}$ ſolid feet; and the ſmall ſized men, or thoſe between 5 feet 3 inches, and 5 feet 6 inches in height, weigh about 135 pounds, and are in bulk equal to about $2\frac{1}{2}$ ſolid feet: And from thoſe expriments it alſo appears, that moſt men are ſpecifically lighter than common water, and much more ſo than ſea water. Conſequently could perſons who fall into water, have preſence of mind enough to avoid the fright uſual on ſuch occaſions, many might be preſerved from drowning: And a very ſmall piece of wood, ſuch as an oar, would buoy a man above water while he had ſpirits to keep his hold.

A GENTLEMAN who had been on board of a Malteſe ſhip of war, obſerved hanging to the tafarel, a block of wood almoſt like a buoy, and ſo ballanced that one end ſwam upright, carrying a little flagſtaff with a ſmall vane; the perſon who was on duty on the poop had orders to cut the rope by which the buoy hung, upon any cry of a perſon's falling over board; and as the block would be in the ſhip's wake by the time the perſon floated therein, he was ſure of having ſomething at hand to ſuſtain him, till the boat could come to his aſſiſtance; and ſhould that take ſo long time to do, as that the diſtance from the ſhip to the man rendered him inviſible, yet the boat would have a mark to row towards, ſhewn them by the vane.

EXAM. IV. *How many ſpars of white fir, each of* 20 *feet long and a foot ſquare, are to be laſhed together, till the raft is ſufficient to float, in common water,* 100 *barrels of gunpowder conducted by four middling ſized men, ſo as to keep the barrels three inches clear of the water?*

A

A BARREL of gunpowder, barrel and all, weighs about 120 ℔.

So 100 barrels will weigh 12000 ℔.

AND 4 men, at 150 ℔ each, weigh 600 ℔.

So that the raft muſt ſuſtain a weight of 12600 ℔.

Now the deal will of it ſelf ſink in the water, until the weight of the water diſplaced is equal to the weight of the wood.

IN each ſpar there is 20 feet of timber.

A CUBIC foot of white deal weighs 35,5624 pounds.

So 35,5624×20=711,248 ℔. the weight of one ſpar.

AND is alſo equal to the weight of the water diſplaced.

A CUBIC foot of common water weighs 62,5 ℔.

THEN 62,5 : 1 : : 711,248 : 11,38 the number of cubic feet which each ſpar will have immerſed by its own weight.

As the barrels are to be 3 inches clear of the water, therefore the ſpar muſt be ſunk 9 inches; and conſequently 15 feet ſolid of each ſpar muſt be immerſed:

THEN 15—11,38=3,62 the additional cubic feet of water to be diſplaced by each ſpar, by its incumbent weight.

AND 1 : 3,62 : : 62,5 : 226,25 ℔. the weight which each ſpar is to ſuſtain.

THEN 226,25 : 12600 : : 1 : 55,6, &c.

CONSEQUENTLY 56 ſuch ſpars laſhed together will make a float ſufficient for to ſuſtain the given weight in the manner propoſed.

ARTICLE X.

Of the quantity of powder uſed in firing of cannon.

ON the circular head of the callipers, on the face D is a table contained between five concentric ſegments of circular rings; the inner one markt GUNS, ſhews the nature of the gun, or the weight of ball it carries:

carries: The two next rings contain the quantity of powder used for proof and service to brass guns; and the two outermost rings shew the quantity for proof and service, used in iron cannon.

The numbers in this table express the English usage, which for the most part, allows the weight of the shot for proof, half its weight in service, and one fourth of its weight of shot for salutes.

The French allowance of powder, for the charge of the piece for service, used to be two thirds of the weight of the shot; twice as much for proof, and one fourth of the weight of shot for salutes.

The Table.

Nature of guns	Brass		Iron		Salutes	Scaling
	Proof	Service	Proof	Service		
Pounders	℔. oz.	℔. oz.	℔. oz.	℔. oz.	℔. oz.	℔. oz.
1	1.0	0.8	1.0	0.8	0.8	0 .1½
1½	1.8	0.12	1.8	0.12	0.12	0 .2
2	2.0	1.0	2.0	1.8	1.0	0 .3
3	3.0	1.8	3.0	1.8	1.8	0 .4
4	4.0	2.0	4.0	2.0	2.0	0 .6
5¼	5.4	2.10	5.4	2.10	2.10	0 .8
6	6.0	3.0	6.0	3.0	3.0	0 .8
8	8.0	4.0	8.0	4.0	3.12	0 .10
9	9.0	4.8	9.0	4.8	4.0	0 .12
12	12.0	6.0	12.0	6.0	4.12	1 .0
18	18.0	9.0	15.0	9.0	6.0	1 .8
24	21.0	12.0	18.0	11.0	7.0	2 .0
26	22.0	13.0	19.0	12.0	7.12	2 .4
32	26.12	16.0	21.8	14.0	9.4	2 .12
36	28.0	18.0	22.0	15.0	10.0	3 .0
42	31.8	21.0	25.0	17.0	11.4	3 .4

Guns carrying ſhot of the weight 1 lb. $1\frac{1}{2}$. lb. 2 lb. 4 lb. $5\frac{1}{4}$ lb. 8 lb. 26 lb. 36 lb. are now out of uſe in the Britiſh navy.

The uſe of this table is obvious: For ſeek the name of the gun in the inner ring, and the weights of powder for proof and ſervice will be found between the ſame two ſtrait lines, like radii; and in one of the other rings, according as it is tituled at the end.

Thus to a braſs 9 pounder, there is allowed 9lb. of powder for to prove, or try the goodneſs of the gun when it is firſt caſt; and 4 lb. 8 oz. of powder for each charge in common ſervice: But an iron 9 pounder has 9 lb. for proof, and 6 lb. for ſervice.

When cannon are proved they are uſually loaded with two ſhot.

On ſhip board, after there are five or ſix rounds fired on warm ſervice, the allowance of powder is to be proportionally leſſened each time the gun is loaded, until the charge is reduced to one third of the weight of the ſhot: And the guns as they grow warm in firing, are not to be wetted leſt the gun be in danger of ſplitting by checking the metal with cold water.

The ingenious Mr. *Robins*, from ſome hints he gathered from a manuſcript lent him by the Right Honourable Lord *Anſon*, adviſes to leſſen conſiderably the common charges allowed to cannon in ſervice: For from thoſe papers it appeared that in ſervice, where 24 pounders have been uſed to batter in breach, the charge was only 8 pounds of powder: Indeed the velocity of the ball could not be quite ſo great with 8 pounds of powder as with 12, and conſequently the ſhot would not be drove ſo far into the rampart, and the breach not made altogether ſo ſoon; notwithſtanding which, the advantages attending the ſmaller charges, greatly overbalanced the difference of a few hours in making a ſufficient breach.

In ſea ſervice it would perhaps be found of greater uſe to begin with one third of the weight of ſhot in

powder, and to diminiſh that to one fourth or one fifth as the gun waxed warm; for by ſome experiments it has appeared, that ſuch ſmall charges of powder has produced greater ravage in timber, than has been found with the uſual charges: From whence it may be reaſonably concluded, that if a ſhot has juſt force enough to go through one ſide of a ſhip, there will be a greater quantity of ſplinters rent out of the plank, and conſequently do more miſchief, than if the ſhot went with a velocity ſufficient to drive it through both ſides of the ſhip.

ARTICLE XI.

Of the number of ſhot or ſhells in a finiſhed pile.

IRON ſhot and ſhells are uſually piled up by horizontal courſes into a pyramidal form, the baſe being either an equilateral triangle, or a ſquare; or a rectangle; in the triangle and ſquare, the pile finiſhes in a ſingle ball; but in the rectangle, the finiſhing is a ſingle row of balls.

IN the triangular and ſquare piles, the number of horizontal rows, or the number counted on one of the angles from the bottom to the top, is always equal to the number counted on one ſide, in the bottom row.

IN triangular piles, each horizontal courſe is a triangular number, produced by taking the ſucceſſive ſums of the numbers 1 and 2; 1, 2 and 3; 1, 2, 3 and 4; 1, 2, 3, 4 and 5, &c. Thus.

Numbers in order	1	2	3	4	5	6	7	8	9	10	11,	&c.
Triangular numb.	1	3	6	10	15	21	28	36	45	55	66,	&c.

AND the number of ſhot in a triangular pile is the ſum of all the triangular numbers taken as far, or to as many terms, as the number in one ſide of the bottom courſe.

A rule to find the number of ſhot in a triangular pile.

COUNT the number in the bottom row, and multiply that

that number more two by that number more one: Then the product multiplied by one ſixth of the ſaid number, the product will be the ſum of all the ſhot in the pile.

EXAM. I. *How many ſhot are in a finiſhed triangular pile, in one ſide of whoſe bottom courſe are 20 ſhot?*

Now the number more two is 22; and the number more one is 21.

AND 22×21 gives 462.

THEN $462 \times \frac{20}{6} = 1540$, the number of ſhot in that pile.

EXAM. II. *Required the number of ſhot in a finiſhed pile; there being in one ſide of the triangular baſe 40 ſhot?*

HERE the number more two is 42; and the number more one is 41.

AND 42×41 gives 1722.

THEN $1722 \times \frac{40}{6} = 11480$ ſhot in that pile.

IN ſquare piles, each horizontal courſe is a ſquare number, produced by taking the ſquare of the number in its ſide.

Number in the ſide	1.	2.	3.	4.	5.	6.	7.	8.	9.	10,	&c.
Squares, or horiz. courſes	1.	4.	9.	16.	25.	36.	49.	64.	81.	100,	&c.

AND the number of ſhot in a ſquare pile is the ſum of all the ſquares, taken from one, as far as the number in the ſides of the bottom courſe.

A rule to find the number of ſhot in a ſquare pile.

COUNT the number in one ſide of the bottom courſe; to that number add one, and to its double add one; multiply the two ſums together; then their product being multiplied by one ſixth of the ſaid number, the product will give the number of ſhot contained in that pile.

EXAM.

Exam. III. *How many ſhot are in a ſquare finiſhed pile, one ſide of its baſe containing 20 ſhot?*

Here the number is 20.

The number more one is 21; and its double, more one is 41.

The product of theſe numbers is 861 ($=21\times41$)

Then $861\times\frac{20}{6}=2870$, the number of ſhot in that pile.

Exam. IV. *Required the number of ſhot in a ſquare finiſhed pile, one ſide of the lower courſe, or tier, having 40 ſhot in it?*

Here the number counted is 40.

That number more one is 41; its double, more one is 81.

And $81\times41=3321$ the product.

Then $3321\times\frac{40}{6}=22140$ the number in that pile.

From theſe examples it may be obſerved, that where room is wanted, 'tis moſt convenient to have the ſhot ſtowed in triangular piles: For on the equilateral triangle, which is leſs than half the area of a ſquare on one of its ſides, there can be piled a greater number than half of what can be raiſed on the ſquare: Indeed the height of a ſquare pile is ſomewhat leſs than a triangular one, as a ſhot will ſink lower in the ſpace between 4 others, than in the ſpace between 3 others, all the ſhot being of equal diameter; they being ſo reckoned in every pile.

In rectangular piles, each horizontal courſe is a rectangle, the upper one being one row of balls: Now every ſuch oblong pile may be conſidered as conſiſting of two parts, one a ſquare pyramid, and the other a triangular priſm.

To find the number of ſhot in a rectangular pile.

1ſt. Take the difference between the number in length and breadth in the bottom courſe.

2d.

2d. MULTIPLY the number in breadth, more one, by half the breadth; the product multiplied by the said difference, will give the number in the prismatic pile.

3d. UPON the square of the breadth, find (by the last rule) the number in a pyramidal pile.

4th. THEN the sum of these two piles will shew the number in the rectangular pile.

N. B. The number of horizontal courses, or rows, is equal to the number in breadth at bottom: And the number less one, in the top row, is the difference between the number in length and breadth at bottom.

EXAM. V. *How many shot are in a finished pile of* 20 *courses, the number in the top row being* 40?

HERE 39 is the difference between the length and breadth.

AND 20 is the breadth.

Now $20+1=21$; and $2\times20+1=41$.

THEN $21\times41\times\frac{20}{6}=2870$, are the shot in the pyramidal pile.

Again. THE breadth more one is 21; and 10 is the half breadth.

AND $21\times10=210$.

THEN $210\times39=8190$, are the shot in the prismatic pile.

CONSEQUENTLY the sum of 2870 and 8190, or 11060 shot will be the number contained in that rectangular pile.

IF any of these piles are broken, by having the upper part taken off, and the remaining number of shot are required; it may be obtained by computing what the whole finished pile would contain; and also what the pile wanting, or taken away contained; for then their difference will shew the number remaining.

THE foregoing rules are thus expressed on the Calipers.

NUMBER of shot or shells in a pile.

LET

Let $n = N^{\circ}$ in an angular row
$m = N^{\circ}$ less one in the top row } of a Pile.

Then $\overline{n+2} \times \overline{n+1} \times \frac{n}{6} = N^{\circ}$ in a $\triangle$
And $\overline{n+1} \times \overline{2n+1} \times \frac{n}{6} = N^{\circ}$ in a $\square$
Also $\overline{\overline{2n+1}+3m} \times \overline{n+1} \times \frac{n}{6} = N^{\circ}$ in a $\sqsubset\!\sqsupset$ } Pile.

In Examples I & III. The letter *n* stands for 20.
And Examples II & IV. The letter *n* stands for 40.
In Example V. The letter *n* stands for 20.
And the letter *m* stands for 39.

Then $2n+1 = 2 \times 20 + 1 = 41.$
And $3m = 117.$
So $2n+1+3m = 158.$
Also $n+1 = 21.$
And $\frac{n}{6} = \frac{20}{6}$
Then $\overline{\overline{2n+1}+3m} \times \overline{n+1} \times \frac{n}{6} = 158 \times 21 \times \frac{20}{6} = 11060.$

ARTICLE XII.

Concerning the fall of heavy bodies.

When heavy bodies are suffered to fall, it is well known they fall in lines perpendicular to the surface of the earth.

The force with which any body in motion strikes an obstacle, depends on the weight of that body, and on the velocity or swiftness with which it moves.

Thus a man by throwing, with the same strength, a pound of iron and a pound of cork, will hit a much harder stroke with the iron than with the cork.

Also a man and a boy each throwing a pound of iron against the same object, the stroke given by the man will be stronger than that given by the boy, on account of the man's weight flying the swiftest.

The same heavy body by falling from different heights, will strike blows of different strength, that being the strongest where the height is greatest. Consequently heavy bodies by falling acquire velocities greater and greater according to the length of their fall.

The three following propositions in falling bodies have been proved many ways.

1st. *That the velocities acquired are directly proportional to the times.*

2d. *That the spaces fallen through are as the squares of the times, or as the squares of the velocities.*

3d. *That a body moving uniformly with the velocity obtained by falling through any height, will fall twice as far in the same time it was passing through that height.*

Experiments shew that heavy bodies fall about 16 feet in one second of time: Consequently at the end of the first second of time, a falling body has acquired a velocity that would carry it down 32 feet in the next second of time.

Then from the foregoing three propositions may be derived the following rules.

1st. That the square root of the feet in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall.

2d. That the square root of the feet in the space fallen through, will ever be equal to four times the number of seconds of time the body has been falling.

3d. And that four times the number of seconds of time in which the body has been falling, is equal to one eighth of the velocity in feet per second, acquired at the end of the fall.

From these three rules most of the questions relative to the fall of bodies may be readily solved.

As

As these rules cannot, for want of room, be put in words at length on the callipers, they are, on the face A of one of the legs, expressed in an algebraic manner. Thus,

FALL OF BODIES.

LET s=space run in feet.
T=time in seconds.
V=velocity in feet per second.

THEN $\sqrt{s}=4T=\frac{1}{8}V$.

BODIES fall 16 feet in 1st sec.

Note. The character $\surd$, signifies the square root of the letter joined to it.

SOME USES.

EXAM. I. *How many feet will a bullet fall in 5 seconds of time?*

HERE the time $T=5$;
THEN 4T, makes $4\times5=20$.
Now $\sqrt{s}=(4T=)\ 20$.
AND $s=(20\times20=)\ 400$.

EXAM. II. *From what height must a bullet fall to acquire a velocity of 160 feet per second?*

THE rule is $\sqrt{s}=\frac{1}{8}V$.
HERE V is 160 feet.
AND $\frac{1}{8}V=(\frac{160}{8}=)\ 200$.
THEN $s=(20\times200=)\ 400$ feet.

EXAM. III. *How long must a bullet be in falling to acquire a velocity of 160 feet per second.*

THE rule is $4T=\frac{1}{8}V$.
HERE $V=160$ feet.
AND $\frac{1}{8}V=(\frac{160}{8}=)\ 20$.
So $4T=20$.
THEN $T=(\frac{20}{4}=)\ 5$ seconds of time.

Exam. IV. *How many ſeconds will it require for a heavy body to fall through a ſpace equal to 3375 yards?*

The rule is $4T = \sqrt{S}$.

Therefore $T = \frac{1}{4}\sqrt{S}$.

Here $S = 3375$ yards, or 10125 feet.

And the ſquare root of 10125 is 100,6.

Then 100,6 divided by 4 gives 25,15.

So that it will require 25″. 9‴ of time for the body to fall through 3375 yards.

ARTICLE XIII.

Rules for the raiſing of water.

Experiments have ſhewn, that taking horſes and men of a moderate ſtrength, one horſe will do as much work in raiſing of water, and ſuch like labour, as five men can.

It has been alſo found, that one man in a minute, can raiſe a hogſhead of water 12 feet high upon a mean: For a ſtout man, well plied with ſtrong liquor, will raiſe a hogſhead of water 15 feet high in a minute: Now as the quantity of liquor equal to a hogſhead was raiſed to theſe heights only by way of experiment for a few minutes, ſuch numbers ought not to be eſteemed as the common labour of a man who is to work 4 or 5 hours on a ſtretch: But it may be reckoned, that of common labouring men, taken one with another, one of them will raiſe a hogſhead of water to 8 feet in height in one minute, and work at that rate for ſome hours.

It is quite indifferent in what manner the man is ſuppoſed to apply his force; whether by carrying the water in manageable parcels up a ſtair-caſe, or raiſing it by means of ſome machine: For the advantages gained by uſing of engines ariſes chiefly from the eaſe with which the power can be applied.

On

On the face A of the callipers, are tne rules, thus denoted.

To raiſe water.

The power $=$ P men.

Or to $\frac{1}{5}$P horſes

Can raiſe to 8P feet high $=$F.

The quantity H, hhds. in T min.

Or G, gallons in 60 T ſeconds.

Or, $H \times F = P \times 8 \times T$ minutes.

N. B. Cubic feet $\times 6,1277$ gives gall.

Here hogſheads are reckoned at 60 gallons, this eſtimate being nice enough for any computations on water engines.

SOME USES.

Examp. I. *How many hogsheads can ſix horſes raiſe, by an engine, to 25 feet high in 3 hours?*

Now 6 horſes, at 5 men to a horſe, is equal to 30 men.

And the time 3 hours is equal to 180 minutes.

The height to be raiſed is 25 feet.

The general rule is $H \times F = P \times 8 \times T$.

Here $F = 25$; $P = 30$; $T = 180$.

And H is required.

$$\text{Then } H = \frac{P \times 8 \times T}{F}$$

$$\text{Or } H = \left(\frac{30 \times 8 \times 180}{25} = \right) 1728 \text{ hogſheads.}$$

Hence this rule. Multiply eight times the power by the time, the product divided by the height, gives the hogſheads.

Examp. II. *It is propoſed to throw out of a pond, by an engine, 432 tuns of water in 3 hours by ſix horſes; to what heighth can the water be raiſed?*

As 4 Hhds make one tun; ſo 432 tuns make 1728 Hhds.

And 3 hours, or 180 minutes is the time.

Also the power of ſix horſes, is equal to that of 30 men.

The general rule is $H \times F = P \times 8 \times T$.

Here $H = 1728$; $P = 30$; $T = 180$.

And F is required.

Then $F = \dfrac{P \times 8 \times T}{H}$

Or $F = \left(\dfrac{30 \times 8 \times 180}{1728} = \right) 25$ feet high.

Hence this rule. Multiply eight times the power by the time; the product divided by the hogſheads, gives the height in feet.

Examp. III. *How long will it require ſix horſes to raiſe with an engine* 1728 *hogſheads of water to the height of* 25 *feet?*

Now the power 6 horſes, is equal to that of 30 men.

The hogſheads to be raiſed are 1728.

The height raiſed to is 25 feet.

The general rule is $H \times F = P \times 8 \times T$.

Here $H = 1728$; $F = 25$; $P = 30$.

And T is required.

Then $T = \dfrac{H \times F}{P \times 8}$

Or $T = \left(\dfrac{1728 \times 25}{30 \times 8} = \right)$ 180 minutes, or 3 hours.

Hence this rule. Multiply the hogſheads, by the height in feet; the product divided by 8 times, the power will give the time in minutes.

Examp. IV. *How many horses will it require to work an engine, to raise 1728 hogsheads to the height of 25 feet, in 3 hours?*

Now the hogsheads to be raised are 1728.
The height to be raised is 25 feet.
The time to be done in is 3 hours, or 180 minutes.
The general rule is $H \times F = P \times 8 \times T$.
Here $H = 1728$; $F = 25$; $T = 180$.
And P is required.

Then $P = \dfrac{H \times F}{8 \times T}$

Or $P = \left(\dfrac{1728 \times 25}{8 \times 180} = \right)$ 30 men, or 6 horses.

Hence this rule. Multiply the number of hogsheads, by the height in feet; the product divided by 8 times the number of minutes, gives the number o men.

ARTICLE XIV.

Of the shooting in cannon and mortars.

It has been proved by many writers, that the flight of shot, or the track they describe in the air, is a curve line called a parabola: But then they suppose that the resistance made by the air is so inconsiderable as scarcely to affect the motion of heavy bodies.

Upon this supposition then, which is very far from being true; there have been collected the following observations and rules.

I. All bodies projected by any force; are urged with two motions, viz. one in the direction of the power exerted by the engine, and the other in a perpendicular direction to the earth, by the force of gravity; and the track or path described by the body with these two forces is a curve called the parabola.

II. The axis of the curve will be at right angles to the horizon; and the part in which the body descends will be alike to that in which it ascended.

III. If the point to which the body arrives in its descent, be on the same level with the point from which it was projected, those points are equally distant from the vertex, or highest point of the curve.

IV. If a body be projected oblique to the horizon, it will fall there again in the same obliquity, and with the same velocity it was projected withal.

V. The horizontal ranges of equal bodies, when projected with the same velocity, at different elevations, will be in proportion to one another; as the right sines of twice the angles of elevation.

VI. Among equal bodies, projected with equal velocities, the heights to which they will rise in the air, are in the same proportion to one another as the versed sines of twice the angles of elevation.

VII. When equal bodies are projected with equal velocities, the times of their continuance in the air will be in proportion to one another as the right sines of the angles of elevation.

VIII. In the same piece, different charges of equally good gunpowder will produce velocities, nearly in the same proportion as the square roots of the weights of the charges.

IX. If equal bodies be projected at the same elevation, but with different velocities, the horizontal ranges will be in proportion to one another, as the squares of the velocities given to the shot, or as the weights of the charges of powder nearly.

X. The greatest horizontal range is double to the height from which the body should fall to acquire that force or velocity which would project it to that horizontal range.

XI. The greatest horizontal range, or distance to which a body can be thrown, will be obtained when

when it is projected at an angle of 45 degrees of elevation.

XII. The greatest height to which a projected body can rise, at an elevation of 45 degrees, is equal to one fourth part of its horizontal range.

XIII. To hit an object that lies above or below the horizon of the piece, the best elevation, is equal to the complement of half the angular distance between the object and the zenith.

XIV. At elevations equally distant from 45 degrees, both above and below it, the horizontal ranges will be equal.

XV. The time which a heavy body, projected at an elevation of 45 degrees, will continue in the air, before it arrives at the horizon, will be equal to the time that body would take to descend, by the force of gravity, through a space equal to the horizontal range.

It has been found that a 24 pounder at an elevation of 45 degrees, and charged with 16 pound of powder, has ranged its shot upon the horizontal plane about 6750 yards.

Therefore 3375 is the impetus, or perpendicular space which a 24 pounder must fall through to acquire such a velocity, as, at an elevation of 45 degrees, would project or throw that shot on the horizon to the distance of 6750 yards.

Now a heavy body falling by the force of gravity through a space equal to 3375 yards or 10125 feet, will, at the end of the fall, acquire a velocity of 804,8 or about 805 feet per second (as shewn at Art. XII.)

And to fall through a space of 805 feet, it would require 25″. 9‴ of time.

The chief of the above principles are shortly expressed on the face B of the callipers in the following manner.

RULES

RULES FOR SHOOTING.

Hor. ranges, as right } ſines of twice } angles of
Heights, are as verſed } elevation.
Time in air, as right ſines of---————}

Impetus $=\frac{1}{2}$ }
Height $=\frac{1}{4}$ } Hor. range, at 45 deg. of elevation.

In aſcents or deſcents, for the beſt elevation.

Take the complement of $\frac{1}{2}$ the angular diſtance from object to zenith.

To apply theſe rules to the practice of ſhooting, it is to be underſtood that the gunner ſhould make an experiment with every gun he has the care of at ſome elevation, ſuppoſe at 45 degrees, and with the uſual charge of powder, and then knowing how far the piece has ranged the ſhot on the horizontal plane; he may apply the reſult of thoſe experiments to other elevations and quantities of powder.

Examp. I. *Suppoſe the greateſt horizontal range to be* 6750 *yards: How far will the ſame piece, and with an equal charge of powder, range a ſhot at an elevation of* 25 *degrees?*

With equal charges the horizontal ranges are as the right ſines of twice the angles of elevation.

Then, As radius, or the ſine of twice 45°
Is to the ſine of 50°, or the ſine of twice 25°,
So is the greateſt horizontal range 6750 yds
To the horizontal range required. 5170 yds.

That is, The extent on the line of ſines from 90° to 50°.

Will on the line of numbers reach from 6750 to 5170.

Examp. II. *The greateſt horizontal range of a* 24 *pounder being* 6750 *yards: To what height will that ſhot riſe at an elevation of* 25 *degrees?*

At

AT an elevation of 45°, the ſhot will riſe $1687\frac{1}{2}$ yards, $=\frac{1}{4}$ of 6750.

AND the heights are as the verſed ſines of twice the angles of elevation.

THEN, As the verſed ſine of 90 degres, or of twice 45°.
Is to the verſed ſine of 50 degrees, or of twice 25°.
So is the height of an elevation of 45°, viz. $1687\frac{1}{2}$.
To the height at an elevation of 25°. 602,8 yards.

THE logarithm verſed ſines on the callipers are the ſupplements of the real verſed ſines; therefore in the uſing of this line the ſupplements of the double angles are to be uſed.

THEN the extent from the verſed ſine of 90° to the verſed ſine of 130° (the ſupplement of 50°) will on the line of numbers reach from $1687\frac{1}{2}$ to 603.

Or thus. TAKE $1292\frac{1}{2}=\frac{1}{4}$ of 5170, the horizontal range on an elevation of 25°.

THEN. The extent on the log, tangents from radius to 25°, will on the line of numbers reach from $1292\frac{1}{2}$ to about 603 yards.

EXAMP. III. *At an elevation of 25 degrees, how many ſeconds will a 24 pounder continue in the air before it arrives at the horizon?*

AT 45° elevation the ſhot takes $35\frac{1}{2}$ ſeconds in the air *.

And the times in air are as the right ſines of the elevations.

THEN As the ſine of the elevation 45 degrees
Is to the ſine of the elevation 25 degrees
So is the time in air at 45°, viz. $35\frac{1}{2}$ ſeconds
To the time in air at 25°, viz. 21. ſeconds.

ON the line of log. ſines take the extent from 45 degrees to 25 degrees; then will this extent, applied to the ſcale of log. numbers, reach from $35\frac{1}{2}$ to 21 ſeconds.

* This time of $35\frac{1}{2}$ ſeconds is derived from Rule XV. Then working by the rules belonging to article XII. it will be found that a heavy body will require $35\frac{1}{2}$ ſeconds to fall through the ſpace of 6750 yards.

And hence may be estimated the lengths of fuses proper for shells to be fired at given elevations and ranges.

Examp. IV. *Required the elevation necessary to strike an object on the horizon at 5170 yards distance, the greatest random of that piece being 6750 yards?*

Say. As the greatest random, 6750 yards
To a proposed random, 5170 yards,
So is radius, or twice the sine of 45 degrees,
To double the elevation required, viz. 50 deg.

The half of which, or 25 degrees, is the elevation necessary to be given to the piece.

This elevation is called the lower one.

And the upper elevation, is at 65 degrees.

For 25 degrees and 65 degrees are equally distant from 45 degrees.

Examp. V. *At an elevation of 45 degrees, 16 ℔. of powder will throw a 24 pounder 6750 yards: How much powder will throw the same shot 5170 yards at the same elevation?*

By rule IX. The charges of powder are nearly as the horizontal ranges.

Then As the horizontal range 6750
To the horizontal range 5170,
So is the given charge 16 ℔.
To the required charge 12,26 ℔.

This proportion may be accurately enough worked by the line of numbers.

For the extent from 6750 to 5170, will reach from 16 to $12\frac{1}{4}$.

Examp. VI. *At an elevation of 25 degrees, a 24 pounder was ranged on the horizon 5170 yards: Required the impetus that would have given an equal velocity to that shot?*

With an equal charge of powder used at 45 degrees of

of elevation, as was used at 25 degrees, the shot would have the greatest horizontal range.

AND with equal charges in the same piece, the impetus is the same at any elevation.

CONSEQUENTLY, to solve this question nothing more is required than to find the greatest horizontal range, which is double to the impetus.

THEN from rule V, by inversion

As the sine of 50 deg. twice the given elevation,
Is to radius, or the sine of twice 45°,
So is the given horizontal range 5170
To the greatest horizontal range 6750,
THE half, or 3375 is the impetus required.
THAT is, the extent on the line of sines from 50° to 90°
WILL on the line of numb reach from 5170 to 6750.

EXAMP. VII. *Suppose the horizontal range of a piece to be 6750 yards: Required the angle of elevation proper to strike an object 12° above the level of the piece, the horizontal distance of that object being 4680 yards?*

SAY, As the greatest horizontal range 6750
Is to the given horizontal distance 4680
So is the cosine of the object's elevation 78° 00′.
To another sine —— — — —42° 42′.

THUS, the extent on the line of numbers from 6750 to 4680

WILL on the line of log. sines reach from 78° to about $42\frac{3}{4}$.

Now on the natural sines, take the extent of $42\frac{3}{4}$ deg.

THEN this extent applied from the natural sine of the elevation 12°

WILL give the natural sine of about $62\frac{1}{2}$ degrees, whose cosine is about $27°\frac{1}{2}$.

OR rather 27°. 37′. its half is 13°. 48′.

THE sum of 90° and the given elevation 12° is 102; the half is 51°.

THEN the sum of these halves ($51° + 13°.48′ =$) 64° 48′, is the greater elevation.

AND

And the difference of theſe halves ($51^\circ - 13^\circ.\ 48=$) $37^\circ.\ 12'$. is the leſſer elevation.

So that the piece pointed at either of theſe elevations, with the charge of powder that gave the horizontal range, the object will be ſtruck.

But in all ſhooting on aſcents or deſcents, it is beſt to take the angle between the object and zenith, and get the complement of the half of that angle; then the piece being elevated to that complement, find by trials what charge will reach the object: For on this elevation, a leſs charge of powder will do the buſineſs than on any other elevation.

So in the foregoing example the diſtance of the object from the zenith is 78°,

The half of 78 is 39, and the complement of 39 is 51°, for the beſt elevation.

ARTICLE XV.

Of the line of inches.

This line, the uſe of which is well known, is placed on the edge of the callipers, or on the ſtrait borders of the faces C, D.

ARTICLE XVI.

Of the logarithmic ſcales of numbers, ſines, verſed ſines and tangents.

These ſcales are placed along the faces C, D of the callipers, near the ſtrait edges, and are marked and numbered as is ſhewn in ſection X; ſome of the uſes of theſe ſcales are alſo ſhewn in the XV and following ſections.

AR-

ARTICLE XVII.

Of the line of lines.

The line of lines is placed on the callipers on the faces C, D, in an angular position, tending towards the centre of the instrument; its construction and uses are the same as described in treating of the sector; the reader will find sufficient instructions in the sections XI and XII.

ARTICLE XVIII.

Of the lines of plans or superficies.

These lines lie on the faces C, D, of the callipers, and like the line of lines tend towards the centre of the instrument: They are marked near the ends of the callipers with the word *Plan*, and have the figures 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 1, running towards the centre: Each of these primary divisions is subdivided into ten parts; and each of the subdivisions is also divided into two, or more parts, according to the length of the callipers.

These divisions reckoned from the centre along either leg, are as the square roots of all the whole numbers under 100; and also, of the half numbers: That is, the distance from the centre to the first 1, is as the square root of 1: From the centre to the next division is as the square root of $1\frac{1}{2}$: To the next as 2, the next as $2\frac{1}{2}$, the next as 3, &c.

And the distance from the centre to the second 1, is as the square root of 10; from the centre to the next division is as the square root of $10\frac{1}{2}$; to the next as 11; to the next as $11\frac{1}{2}$, &c. So that the distances from the centre to 2, to 3, to 4, and so on to 10, are as the square roots of 20, 30, 40, and so on to 100; and the intermediate divisions and subdivisions are estimated as before shewn between 1 and 10.

This

This line is easily constructed from a table of the square roots of all the units and half units under 100; together with a scale of the intended length of the line of plans, divided into 500 or 1000 equal parts; and such a scale is the line of lines.

In the followng solutions, the lengths of lines are supposed to be taken between the points of a pair of compasses: And when the callipers are said to be opened to any line; it means, to the distance of the points of the compass between which that line was taken; the points being applied transversely to the legs of the callipers, as shewn for the sector at section XII.

SOME USES OF THE SCALES OF PLANS.

Examp. I. *To find the square root of a given number.*

1st. On the line of plans seek the division representing the given number: Observing, that numbers of an odd number of places are best found between the divisions 1 and 1; and those of an even number of places, between the 2d 1 and the 10 at the end.

2d. Take, with the compasses, the distance between that division and the centre of the callipers; and this extent being applied, from the centre laterally along the *line of lines*, will give the square root of the number proposed.

Thus the square root	of 9	is	3
	of 900	is	30
	of 90000	is	300
	&c.		&c.

The given numbers being reckoned between the two divisions marked 1 and 1.

Again the square root	of 36	is	6
	of 360	is	18,9
	of 3600	is	60
	of 36000	is	189,7.

If the integer places in the given number are even, the root will consist of half as many places: But if the number

number of integers be odd, increaſe it by one, and the integer places in the root will be half that number of places.

Thus numbers of two, four, ſix, eight integer places, will have roots conſiſting of one, two, three, four, &c. places: And numbers conſiſting of one, three, five, &c. places, have roots of one, two, three, &c. places.

Examp. II. *Between two given numbers (ſuppoſe 4 and 9) to find a mean proportional.*

1ſt. Take the greater of the given numbers (9) laterally from the line of lines, and make this extent a tranſverſe diſtance to (9 and 9) the ſame number on the lines of plans.

2d. Take the tranſverſe diſtance between (4 and 4) the leſſer given number on the lines of plans, and this extent applied laterally on the line of lines, will give (6 for) the mean proportional ſought.

For 4 : 6 :: 6 : 9.

By this example it is eaſy to ſee how to find the ſide of a ſquare equal to a ſuperficies whoſe length and breadth are given.

Examp. III. *Two ſimilar, or like, ſuperficies being given; to find what proportion they have to one another.*

1ſt. Take one ſide of the greater ſuperficies between the points of the compaſſes, and make this extent a tranſverſe diſtance on the line of plans between 10 and 10; or 100 and 100: or on any other number.

2*d.* Apply a like ſide of the leſs ſuperficies tranſverſely to the line of plans, and the diviſions it falls on will ſhew the number, that to the former number (taken tranſverſely for the ſide of the greater ſuperficies) bears the ſame proportion of the leſſer ſuperficies to the greater.

This propoſition may be wrought laterally on either of the legs, reckoning from the centre: For like ſides of ſimilar plans being laid from the centre on either

leg, will give numbers ſhewing the proportion of thoſe plans.

EXAMP. IV. *To find the ſides, or other lines, of a ſuperficies* A, *which ſhall be ſimilar to a given ſuperficies* B, *and in a given proportion to* B, *ſuppoſe as* 3 *to* 7?

1*ſt*. To the ſcales of plans, apply tranſverſely, any given line of B to the conſequent of the given ratio, as from 7 to 7.

2*d*. TAKE the tranſverſe diſtance, on the plans, of the given antecedent, as from 3 to 3, and this extent will be a like line of the figure A.

3*d*. As many lines being thus found as is neceſſary, the figure A may be conſtructed.

EXAMP. V. *To find the ſides, or other lines, of a ſuperficies* D, *which ſhall be like to either of two given plane figures* A *and* B; *and alſo be equal to the ſum or difference of* A *and* B.

1*ſt*. FIND (by Ex. 3.) two numbers expreſſing the proportion of the given figures A and B; and take the ſum and difference of thoſe numbers.

SUPPOSE the proportion of A to B, to be as 3 to 7.

THEIR ſum is 10, and their difference is 4.

THEN if D is to be like A.

FOR the ſum, it will be 3 : 10 : : A : D.

FOR the diff. it will be 3 : 4 : : A : D.

BUT if D is to be like B.

THEN, for the ſum, it will be 7 : 10 : : B : D.

AND for the diff. it will be 7 : 4 : : B : D.

2*d*. FIND (by Ex. 4.) the ſides of a ſuperficies D, ſimilar to A, ſo that A may be to D as 3 to 10 for the ſum, or as 3 to four for the difference; or if like to B, ſo that B may be to D as 7 to 10 for the ſum, or as 7 to 4 for the difference.

AND thus, a ſufficient number of lines being found the figure D may be conſtructed.

EXAMP. VI. *Three numbers being given to find a fourth in a duplicate proportion:* Or, *the like sides* a, b, *of two similar figures* A, B, *being known, and also the area* A, *of one, to find the area* B, *of the other.*

ON the scale of plans, take the given superficies A laterally; and on the scale of lines, apply this distance transversely to the given side *a* of that superficies: Take the transverse distance of the given side *b* of the other superficies, from the scale of lines; then this distance applied laterally on the scale of plans, will shew the area of B.

THUS. *If* 40 *poles be the side of a square whose area is* 10 *acres; what is the area of that square whose side is* 60 *poles?*

TAKE the lateral distance 10 on the scale of plans; apply this distance transversely to 40 and 40 on the line of lines: Then the transverse distance of 60 and 60 on the lines, applied laterally to the scale of plans, will give $22\frac{1}{2}$ acres the area required.

AGAIN. *How many acres of woodland measure, of* 18 *feet to the pole, is in that field which contains* 288 *acres, at* $16\frac{1}{2}$ *feet to the pole?*

APPLY the lateral distance of 288, taken from the scale of plans, to the line of lines, transversely from 18 to 18; then the transverse distance of $16\frac{1}{2}$ and $16\frac{1}{2}$ on the lines, will, on the scale of plans, give 242 the area in woodland acres.

EXAMP. VII. *To open the callipers, so that the lines of plans make with one another a right angle?*

ON the line of plans take the lateral extent of any number thereon.

THEN set the callipers so, that this extent shall be a transverse distance to the halves of the former number,

and the lines of plans will then ſtand at right angles to one another.

THUS: The lateral extent of 60 on the plans, put tranſverſely to 30 and 30 on the plans, will ſet thoſe lines at right angles to one another.

ARTICLE XIX.

Of the line of ſolids.

THESE lines are laid on the faces of D, C, of callipers, like ſectoral lines tending to the centre, and are diſtinguiſhed by the letters SOL placed at their ends.

THERE are twelve primary diviſions on theſe lines marked 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; each of the eleven ſpaces or intervals is divided into ten other parts; and each of theſe parts is divided into two or more parts, according to the length of the inſtrument.

THESE diviſions are beſt taken from a ſcale of equal parts, ſuch as the line of lines, and thence transferred to the ſcales of ſolids, reckoning from the centre; from whence the ſeveral diſtances of the diviſions are, as the cube roots of ſuch numbers under 100 as are intended to be introduced.

THUS, the diſtance of the firſt 1 from the centre is as the cube root of $\frac{1}{10}$, and the greater diviſions following to the ſecond 1, expreſs the cube roots of $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c. to the number 1, which the ſecond 1 ſtands for; and if theſe ſpaces are ſubdivided, their diſtances from the centre are as the cube roots of $\frac{15}{100}$, $\frac{25}{100}$, $\frac{35}{100}$, $\frac{45}{100}$, &c.

THE diſtance from the centre to the ſecond 1 is as the cube root of 1, and the greater diviſions between the ſecond 1 and the third 1, are as the cube roots of the whole numbers 2, 3, 4, 5, 6, 7, 8, 9; the intermediate ſmaller diviſions are as the cube roots of the mixed numbers to which they belong: Thus if the ſpace between the diviſions repreſenting the roots of 1 and 2 is parted into 4; then thoſe ſubdiviſions will be

as

as the cube roots of 1 $\frac{25}{100}$, 1 $\frac{5}{10}$, 1 $\frac{75}{100}$; and the like for other ſubdiviſions.

The diſtance between the centre and the third 1 is as the cube root of 10; and ſo the following diviſions marked with 2, 3, 4, &c, to 10, are as the cube roots of 20, 30, 40, &c. to 100: each of theſe ſpaces are divided into 10 parts, which are as the cube roots of the intermediate whole numbers; and if theſe ſubdiviſions are again divided, theſe latter diviſions will be as the cube roots of the mixed numbers to which they belong.

On the French inſtruments, the diviſions of this line is uſually extended to 64; and conſequently only the cube roots of all the integer numbers under 64 are thereon expreſſed: Now whether the diviſions proceed only to 64 or to 100, the beſt way of laying them down is from a table of cube roots ready computed, reckoning the length of the greateſt root, or the length of the ſcale of ſolids, to be equal to the length of the line of lines, taken from the centre.

The cube roots of $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, are, 0,464. 0,585. 0,669. 0,737. 0,794. 0,843. 0,888. 0,928. 0,965.

The following table contains the cube roots of all the whole numbers from 1 to 100.

TABLE *of cube numbers and their roots.*

Cubes	Roots	Cubes	Roots	Cubes	Roots	Cubes	Roots	Cubes	Roots
1	1,000	21	2,759	41	3,448	61	3,936	81	4,327
2	1,260	22	2,802	42	3,476	62	3,958	82	4,344
3	1,442	23	2,844	43	3,503	63	3,979	83	4,362
4	1,587	24	2,884	44	1,530	64	4,000	84	4,379
5	1,710	25	2,924	45	3,557	65	4,021	85	4,397
6	1,817	26	2,962	46	3,583	66	4,041	86	4,414
7	1,913	27	3,000	47	3,609	67	4,061	87	4,431
8	2,000	28	3,036	48	3,634	68	4,082	88	4,448
9	2,080	29	3,072	49	3,659	69	4,102	89	4,465
10	2,154	30	3,107	50	3,684	70	4,121	90	4,481
11	2,224	31	3,141	51	3,708	71	4,141	91	4,498
12	2,285	32	3,175	52	3,732	72	4,160	92	4,514
13	2,351	33	3,207	53	3,756	73	4,179	93	4,531
14	2,410	34	3,240	54	3,780	74	4,198	94	4,547
15	2,466	35	3,271	55	3,803	75	4,217	95	4,563
16	2,520	36	3,302	56	3,826	76	4,236	96	4,579
17	2,571	37	3,332	57	3,848	77	4,254	97	4,595
18	2,621	38	3,362	58	3,871	78	4,273	98	4,610
19	2,668	39	3,391	59	3,893	79	4,291	99	4,626
20	2,714	40	3,420	60	3,915	80	4,309	100	4,642

THE numbers in the foregoing table may be laid on the line of solids in the following manner.

MAKE the length of the line of solids equal to the length of the line of lines, apply this extent transversely to 4,642 on the line of lines; then the other numbers in the table taken transversely from the line of lines, are to be laid laterally, from the centre, on the line of solids.

Some uses of the lines of solids.

EXAMP. I. *To find the cube root of a given number.*

SEEK the given number on the line of solids, and take its extent from the centre.

THEN this extent applied laterally to the line of lines will give the cube root sought. It

It ſhould be remarked, that a given number of
of 1, 2 or 3 places, has a root of one place.
of 4, 5 or 6 places, has a root of two places.
of 7, 8 or 9 places, has a root of three places.

And when a given number is ſought for on the line of ſolids,

The primary diviſions from 1 to 10 may be reckoned either as ſo many hundreds, or as ſo many hundred thouſands, or as ſo many hundred millions.

Thus the diviſion marked 5 may either repreſent 500, or 500000, or 500000000.

And the like of the other primary diviſions and their intermediates.

And hence the diviſions between the centre and the firſt of the primary ones, are to be eſtimated for numbers of one, two, four, five, ſeven and eight places.

Examp. II. *To a number given, to find another in a triplicate ratio of two given numbers.*

Thus. Suppoſe a ſhot of 4 inches diameter to weigh 9℔; required the weight of that ſhot which is 8 inches in diameter?

Here a number is to be found, that to 9 ſhall be in the triplicate ratio of 4 to 8.

That is, as the cube of 4 is to the cube of 8, ſo is 9 to the number ſought.

Now from any ſcale of equal parts, ſuppoſe inches, take 4; and make it a tranſverſe diſtance to 9 and 9 on the line of ſolids (reckoning the 10 at the end, as 100): Then will the extent of 8 inches, applied tranſverſely to the line of ſolids, give 72 for the number ſought, which is the pounds weight of a ſhot of 8 inches diameter.

Again.

Again. *Suppose a ship of* 2000 *tons burthen is* 144 *feet* 6 *inches on the keel, and* 51 *feet by the beam: Required the length and breadth of another similar ship that shall be of* 1415 *tons burthen?*

From any scale of equal parts take $144\frac{1}{2}$ and make this extent a transverse distance to 2000 on the line of solids; then will the transverse distance of 1415 taken on the line of solids give the length of the keel, which applied to the said scale of equal parts will give about $128\frac{3}{4}$ feet.

Also the extent in equal parts of 51 being made a transverse distance to 2000 on the lines of solids; then the transverse distance on the solids of 1415 will give in equal parts $46\frac{1}{3}$ feet for the breadth by the beam.

Examp. III. *Between two given numbers or lines to find two mean proportionals.*

1*st*. From any scale of equal parts take the measure of the greatest of the given lines or numbers, and apply this extent transversely to that number on the line of solids; then the transverse extent on the solids, of the least of the given numbers, being taken, will be the greater of the required means, whose measure will be found on the said scale of equal parts.

2*d*. Make the extent of the greater mean, a transverse distance to the greater of the given numbers, on the line of solids; then the transverse distance of the lesser of the given numbers, taken from the line of solids, will give the lesser of the required means.

Suppose two mean proportionals were required between 9 *and* $41\frac{2}{3}$.

The lateral extent of $41\frac{2}{3}$, taken from the line of lines, apply transversely to $41\frac{2}{3}$ and $41\frac{2}{3}$ on the line of solids; then the transverse extent of 9 and 9 taken on the solids, and applied laterally to the line of lines will give 25 for the greater of the two means.

Apply

APPLY the ſaid extent of 25 tranſverſely to $41\frac{2}{3}$ and $41\frac{2}{3}$ on the line of ſolids; then the tranſverſe extent on the ſolids from 9 to 9 applied laterally to the line of lines, will give 15 for the leſſer mean.

FOR 9, 15, 25 and $41\frac{2}{3}$ are in continual proportion.

EXAMP. IV. *To find the ſide of a cube equal to a parallelopipedon whoſe length, breadth and depth are given.*

1ſt. BETWEEN the breadth and depth find a mean proportional by Ex. 2. Art. 18.

2d. FIND the meaſure of the mean proportional on the line of lines, and apply it to the lines of ſolids tranſverſely, at the numbers expreſſing that meaſure: Then the tranſverſe extent of the length being taken from the line of ſolids and applied laterally to the line of lines, will give the ſide of a cube equal to that parallelopipedon.

THUS, *Suppoſe a parallelopipedon, whoſe length is* 72, *breadth* 64, *and depth* 24.

THE number 64 taken laterally from the line of lines and applied tranſverſely to 64 and 64 on the line of plans; then the tranſverſe diſtance of 24 and 24 on the plans meaſured laterally on the line of lines gives about 39,2 for the mean proportional.

APPLY the extent of the mean proportional, to 39,2 tranſverſely on the line of ſolids; then the tranſverſe extent of 72 and 72 on the ſolids, being applied to the line of lines laterally, will give 48 for the ſide of the cube equal in ſolidity to the given parallelopipedon.

FOR $48 \times 48 \times 48 = 24 \times 64 \times 72 = 110592$.

EXAMP. V. *Two ſimilar ſolids* A *and* B *being given, to find their ratio.*

1ſt. TAKE any ſide of the ſolid A, and apply it tranſverſely

transverſely on the line of ſolids from 10 to 10, or from any other number to its oppoſite.

2*d.* APPLY the like ſide of the ſolid B tranſverſely to the lines of ſolids, and obſerve the number it falls on: Then will the numbers on which thoſe tranſverſe extents fall, ſhew the ratio of the ſolids A and B.

EXAMP. VI. *A ſolid* A *being given to find the dimenſions of a ſimilar ſolid* B, *that to* A *ſhall have any aſſigned ratio.*

1*ſt.* ON the line of ſolids ſeek two numbers expreſſing the terms of the given ratio.

2*d.* TAKE the extent of one ſide of the given ſolid A, and apply it tranſverſely on the lines of ſolids to the antecedent of that given ratio; then the tranſverſe extent of the conſequent taken on the lines of ſolids will be a like ſide of the ſolid B.

THUS. *To find the ſide of a cube* B, *double to a given cube* A.

HERE the ratio is as 1 to 2.

APPLY the ſide of the cube A to the lines of ſolids tranſverſely from 1 to 1; that is from 10 to 10; then will the tranſverſe diſtance of the numbers 2 and 2 or 20 and 20 ſhew the ſide of the cube B.

AGAIN. *To find the diameter of a ſphere* B, *that to the ſphere* A, *whoſe diameter is given, ſhall be in the ratio of 3 to 2.*

MAKE the diameter of the ſphere A a tranſverſe diſtance to 2 and 2 on the lines of ſolids; then will the tranſverſe diſtance of 3 and 3 on the line of ſolids be the diameter of the ſphere B.

EXAMP. VII. *Any number of unequal ſimilar ſolids being given; to find the ſide of a ſimilar ſolid equal in magnitude to the ſum of the magnitudes of the given ſolids.*

TAKE, in equal parts, a number expreſſing the ſide of

of one of the given solids, and apply this extent to the line of solids transversely, to any number (suppose 10 at the 3d 1).

ALSO take in the same equal parts, the numbers shewing the similar sides of the other solids, and apply these extents to the lines of solids transversely, noting the numbers they fall on.

THEN will the transverse extent on the line of solids of a number equal to the sum of the noted numbers, be the like side of the similar solid required, which applied to the same scale of equal parts the others were taken from will give the measure of that side.

THUS. *What will be the diameter of that iron shot cast from 3 other shot whose diameters were 4 inches, 4, 4 inches, and 5 inches; supposing no waste in melting?*

MAKE 4 inches a transverse extent on the line of solids, to any number suppose 10. Then 4,4 inches applied transversely to the solids will give about $13\frac{1}{3}$; and 5 inches also applied transversely to the solids will give about $19\frac{2}{3}$: Now the sum of the noted numbers 10 and $13\frac{1}{3}$ and $19\frac{2}{3}$ will be 43; then the transverse extent of 43 on the line of solids will give $6\frac{1}{2}$ inches for the diameter of the new shot.

EXAMP. VIII. *To find the dimensions of a solid which shall be equal to the difference of two given similar solids, and also similar to them.*

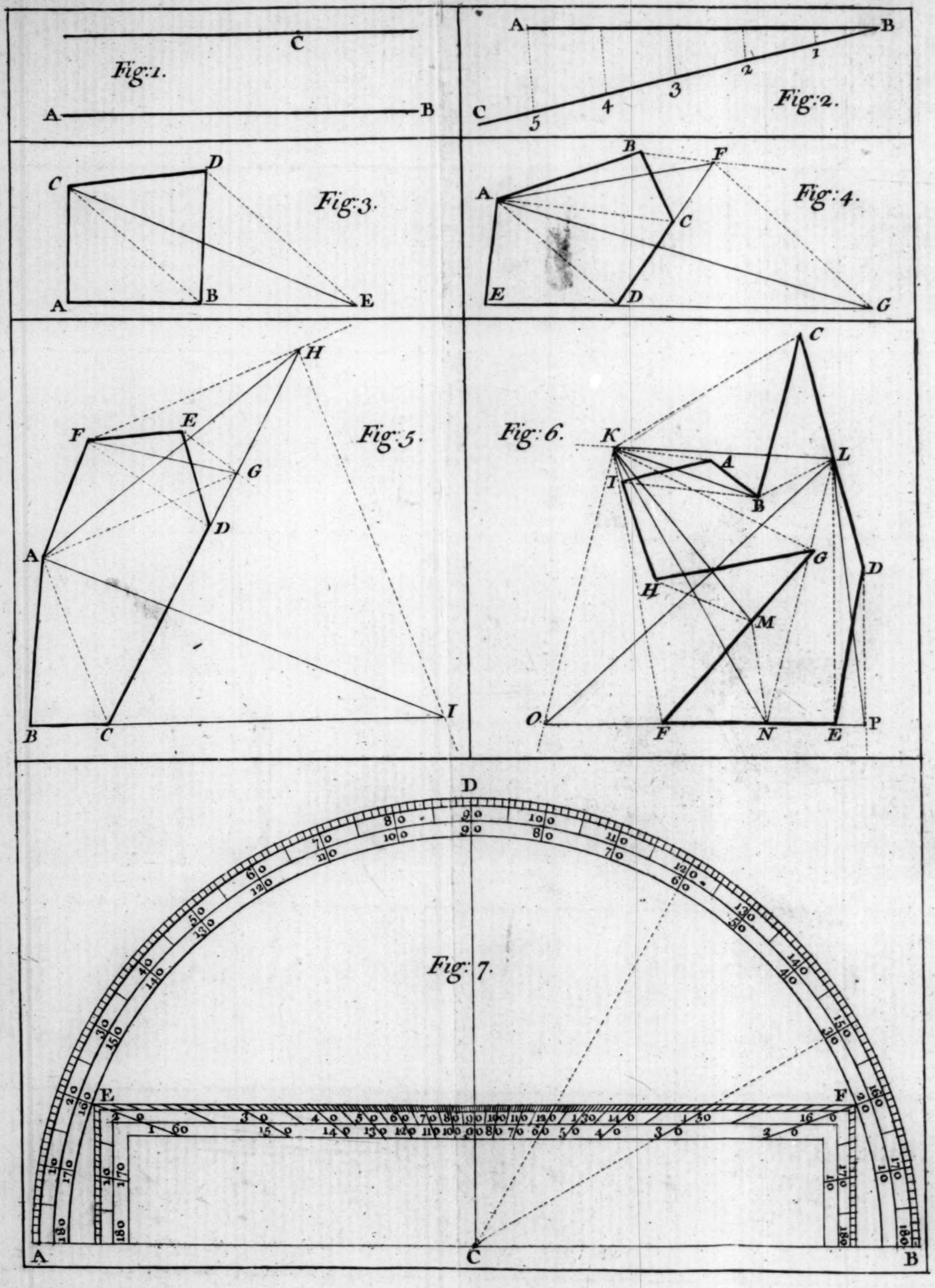
APPLY a dimension of one solid transversely to the line of solids at any number; and also note what number on the line of solids, the like dimension of the other solid falls transversely on; take the difference of those noted numbers; and on the line of solids take transversely the extent of the remainder, and that will be a like dimension of the similar solid required.

THUS.

Thus. *With the powder out of a ſhell of* 10 *inches concave diameter is filled a ſhell of* 7 *inches : What ſized ſhell will the remaining powder fill ?*

The extent of 10 inches being applied tranſverſely to the lines of ſolids, at any number ſuppoſe 100 ; the extent of 7 inches will fall tranſverſely on the lines of ſolids, about the number $34\frac{1}{4}$: The difference between 100 and $34\frac{1}{4}$ is $65\frac{3}{4}$: Then the tranſverſe extent at $65\frac{3}{4}$ on the line of ſolids, will give 8,7 inches for the concavity of that ſhell which the remaining powder will fill.

FINIS.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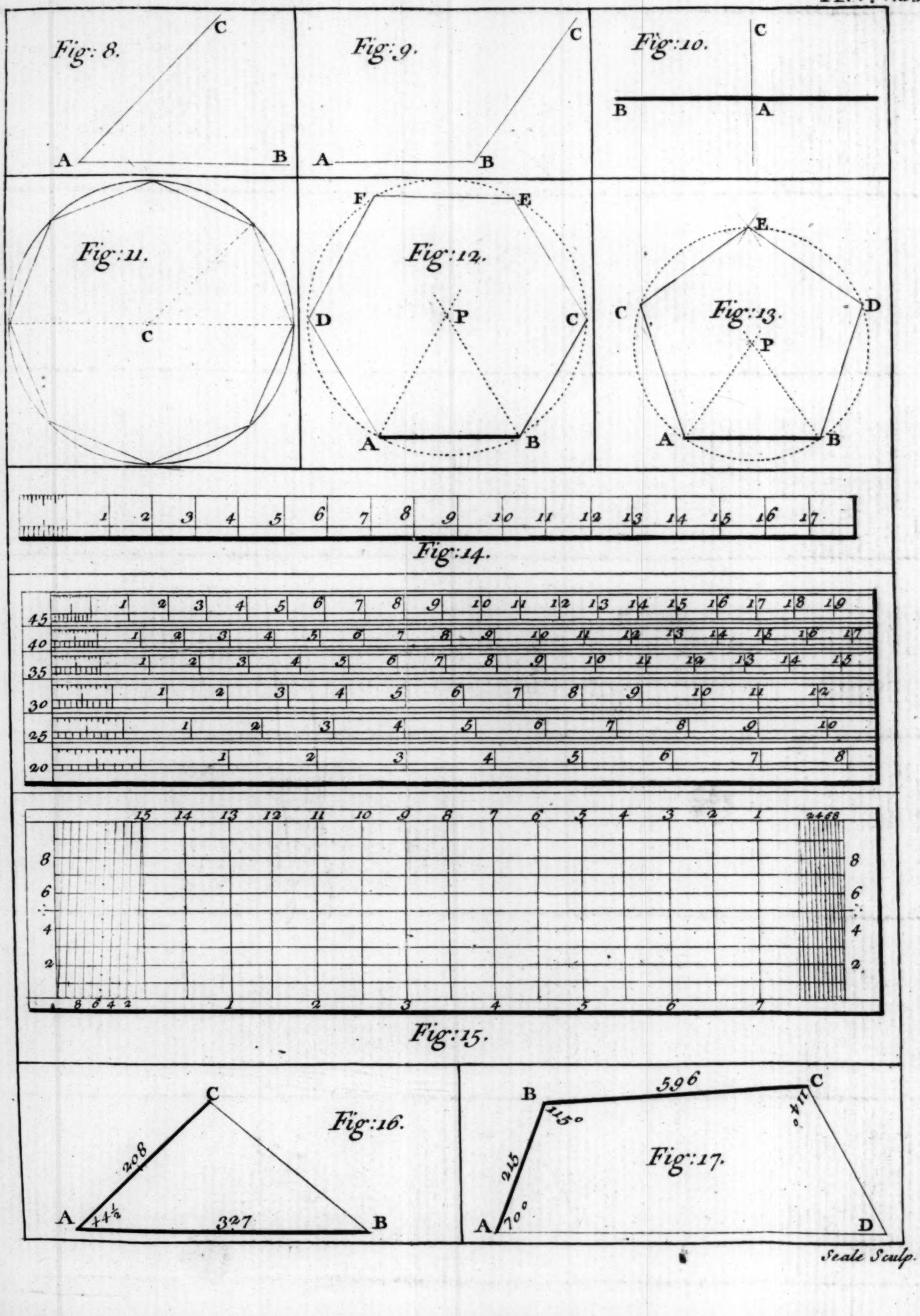
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Fig: 9.
Fig: 10.
Fig: 11.
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Fig: 14.
Fig: 15.
Fig: 16.
208
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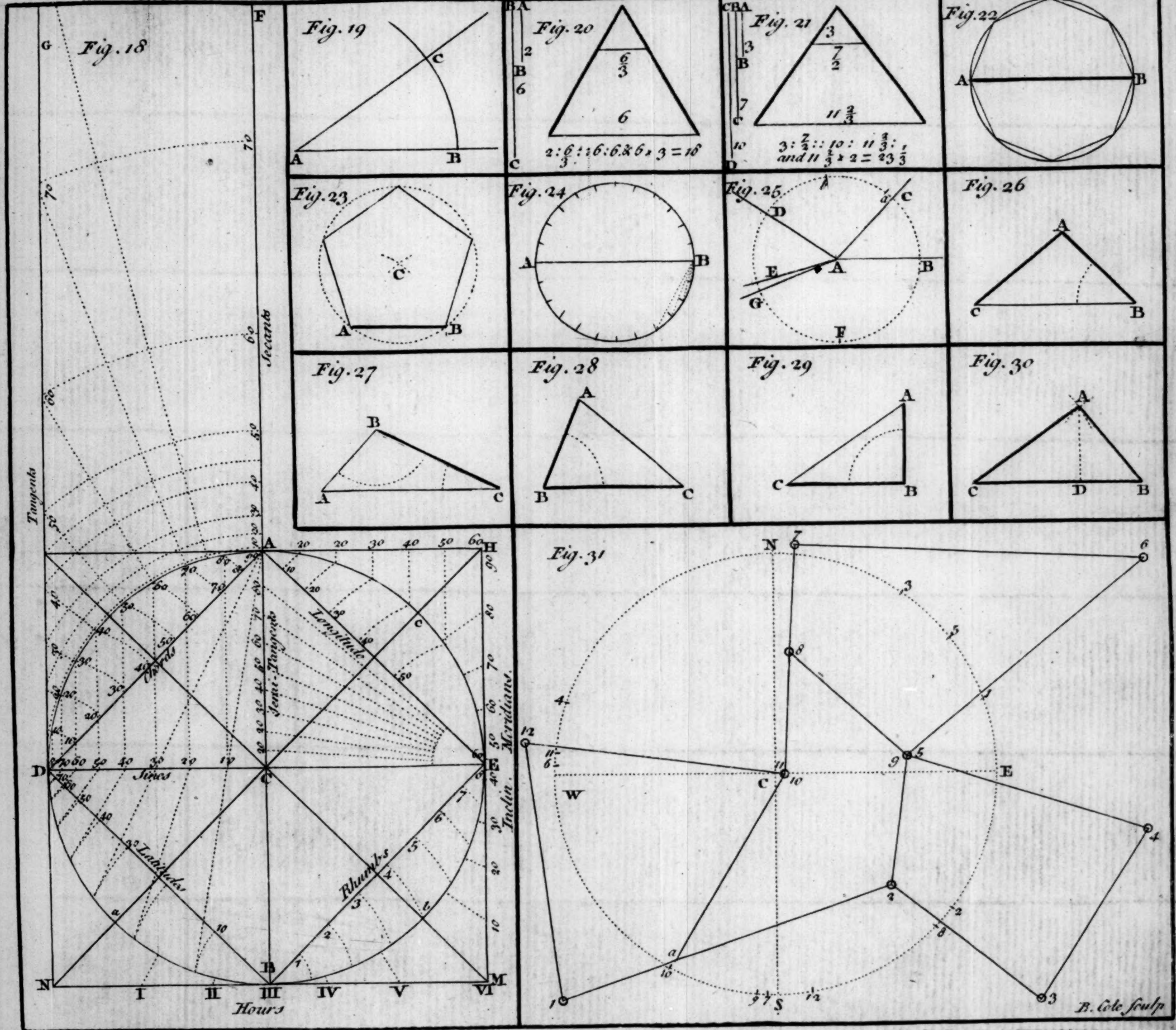
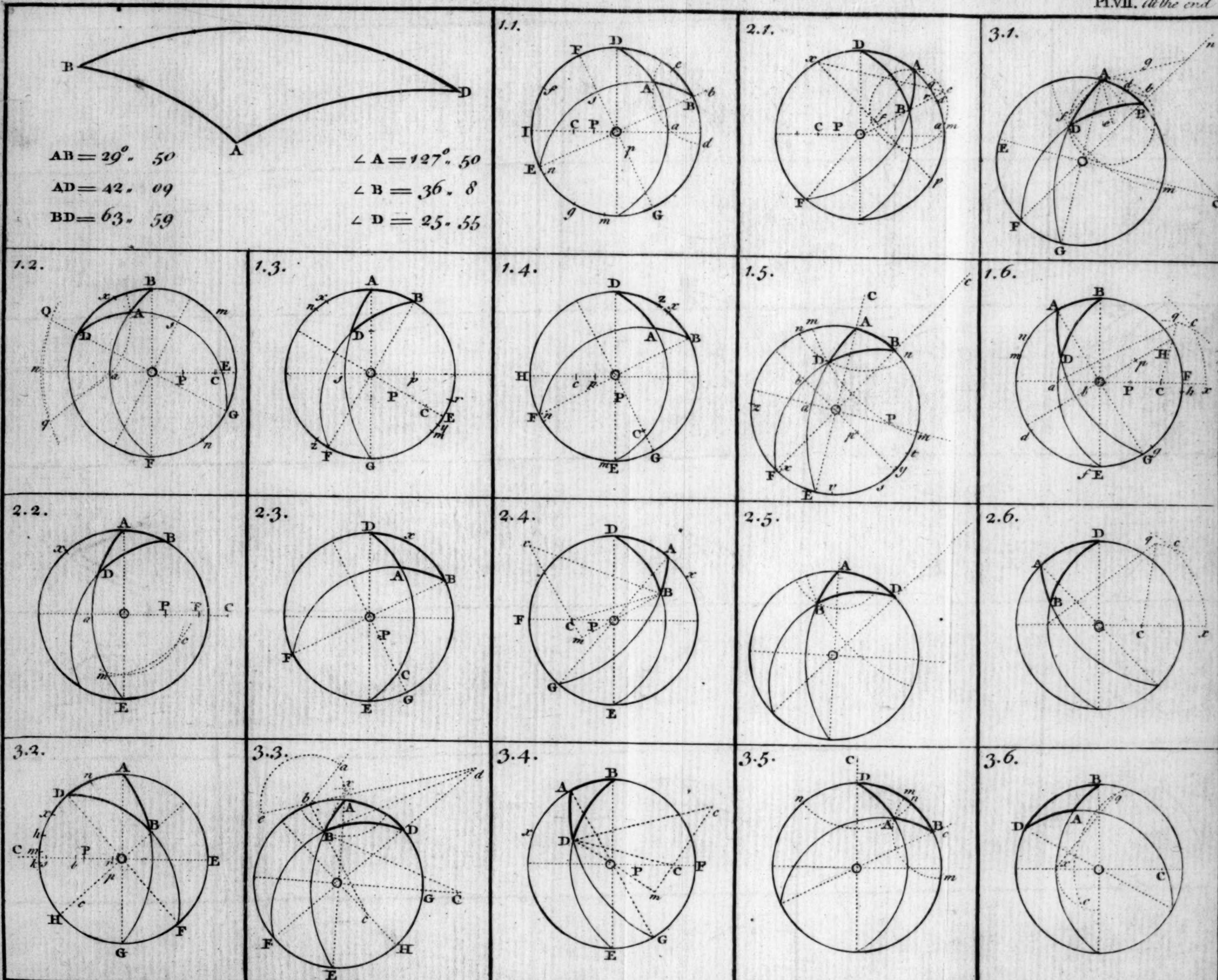
Fig: 17.
596
115°
114°
215
70°

Seale Sculp.

Pl VI at the end.





AB = 29°. 50
AD = 42. 09
BD = 63. 59
∠ A = 127°. 50
∠ B = 36. 8
∠ D = 25. 55
1.1.
2.1.
3.1.
1.2.
1.3.
1.4.
1.5.
1.6.
2.2.
2.3.
2.4.
2.5.
2.6.
3.2.
3.3.
3.4.
3.5.
3.6.

B. Cole sculp.

188

Pl.VIII at end